ELECTRIC TRANSIENTS

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BY

CARL EDWARD MAGNUSSON

AUTHOR OF "ALTERNATING CURRENTS," PROFESSOR OF ELECTRICAL ENGINEERING, DEAN OF THE COLLEGE OF ENGINEERING, DIRECTOR OF THE ENGINEERING EXPERIMENT STATION, UNIVERSITY OF WASHINGTON

A. KALIN

INSTRUCTOR IN ELECTRICAL ENGINEERING, UNIVERSITY OF WASHINGTON

J. R. TOLMIE

INSTRUCTOR IN ELECTRICAL ENGINEERING, UNIVERSITY OF WASHINGTON

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PREFACE

Transient electric phenomena generally increase in commercial importance with the size and complexity of electric systems, and a knowledge of the fundamental principles of electric transients and their application to the solution of quantitative problems is as essential to the successful operation of large power and communication systems as a mastery of the basic laws of direct and alternating currents.

This work is an outline of an introductory lecture and laboratory course given during the past twelve years to electrical engineering students in the University of Washington. The purpose of the book is to aid the student in gaining clear concepts of the fundamental principles of electric transient phenomena and their application to quantitative problems. The course as outlined is professedly of an elementary character with emphasis placed on the physical properties of electric transients. The text is illustrated and supplemented by a large number of oscillograms of transients that occur in the various types of machines and electric circuits in common use in electrical engineering laboratories. The problems are based on quantitative data obtained from laboratory experiments under circuit conditions that may easily be reproduced by the student.

Quantitative laboratory work is essential in order to readily gain insight into the physical nature of transient electric phenomena. It is advisable to require the student to devote at least two-thirds of the time allotted to a course in electric transients to the taking of oscillograms. Adjusting an oscillograph so as to obtain sharply defined, well proportioned oscillograms of electric transients is an effective method for acquiring due appreciation of quanti-

tative values, both absolute and relative, of the factors involved. The quality of the photographic record depends as much on painstaking care in handling the films and in developing and printing the oscillograms as on skilful operation of the oscillograph. Many pitfalls in the photographic part of the work may be avoided by carefully following the directions given in the Appendix.

No attempt is made to give references to original investigations or to papers and books dealing with the various phases of electric transient phenomena, as the principles discussed are well established and the material is arranged in text book form. A distinctive feature of the book lies in the illustrations. All of the oscillograms were taken by A. Kalin and J. R. Tolmie or by students in the course under their direction in the electrical engineering laboratories of the University of Washington.

C. EDWARD MAGNUSSON.

SEATTLE, WASH.,
March, 1922.

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ELECTRIC TRANSIENTS

CHAPTER I

INTRODUCTION

The laws for direct currents, as usually expressed, state the relations of the several factors involved under continuous or permanent conditions, and cannot be correctly applied while the current or voltage is increasing or decreasing. Similarly, alternating currents are expressed as continuous phenomena by means of effective values and complex quantities, on the basis that the successive cycles are of the same magnitude and wave shape. Observations and test data for both the direct-current and alternating-current systems are ordinarily taken only during steady or permanent conditions. The equations derived, and the data obtained from tests, apply only to permanent or constant conditions and cannot be correctly applied during transition periods when the conditions vary. Transient electric phenomena, as the term implies, are usually of short duration and relate to what occurs in an electric circuit between periods of stable conditions. This definition is, however, not rigidly adhered to in electrical discussions. Frequently other disturbances that militate against successful operation of electric systems, such as unstable electric equilibrium, permanent instability, resonance and cumulative oscillations are included with the true transients under the caption of transient electric phenomena.

It is important that the student should realize that electric transients are of very frequent occurrence in all commercial electric systems. Any change, such as the starting or stopping of a motor, the turning on of a lamp, or any change in the operating conditions necessitates a

re-adjustment of the energy content in the whole system and produces electric transients just as truly as a stroke of lightning or a short circuit. In the operation of street car systems the changes in load, and hence the transients on the system, are so frequent that they overlap and occupy by far the greater part of the time; hence, for street railway systems, it might appear simpler to define the permanent or steady conditions as short periods occurring between successive series of overlapping transients.

Electrical engineering deals with the transmission and transformation of electric energy. During permanent conditions the flow of energy is uniform and continuous; any change in the power indicates a transient condition. Changes in the current and voltage factors imply a corresponding change in the energy content of the electric field, since a magnetic field surrounds all electric currents, and an increase or decrease in the current necessitates a corresponding change in the stored magnetic energy. Similarly, any change in voltage between conductors must be accompanied by a corresponding re-adjustment in the energy stored in the dielectric field of the system.

Magnetic Circuit.—In the study of transient phenomena, as well as of all phases of the electric field, Faraday's concept of magnetic and dielectric lines of force is of fundamental importance. All magnetic lines are continuous and closed on themselves. Ohm's law applies to the magnetic circuits in the same way as to the electric circuit. The magnetic flux produced is equal to the magneto-motive force divided by the reluctance.

$$\text{Magnetic flux} = \frac{\text{magneto-motive force}}{\text{reluctance}};$$

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} \text{ or } \mathcal{F} = \mathcal{R}\Phi \tag{1}$$

The magnetic field is produced by, and is proportional to, the electric current.

$$\Phi = Li \tag{2}$$

The proportionality factor L is called the inductance of the circuit.

The reluctance varies directly as the length and inversely as the cross section of the magnetic circuit. The specific reluctance per cm.[3] is the reciprocal of the permeability μ. If the magneto-motive force is expressed in ampere turns, the resultant field intensity is given by the equation.

$$H = 4\pi nI\ 10^{-1} \text{ per cm.} \tag{3}$$

This magnetizing force produces a magnetic flux density of B lines per cm.[2] in materials having μ permeability.

$$B = \mu H \text{ lines per cm.}^2 \tag{4}$$

The permeability is the reciprocal of the specific reluctance in the magnetic circuits and corresponds to the specific

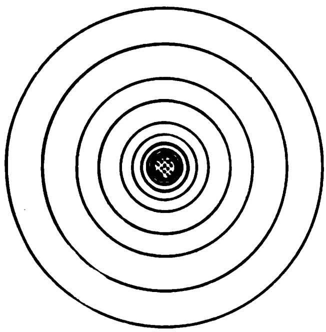

FIG. 1.—Magnetic field of single conductor.

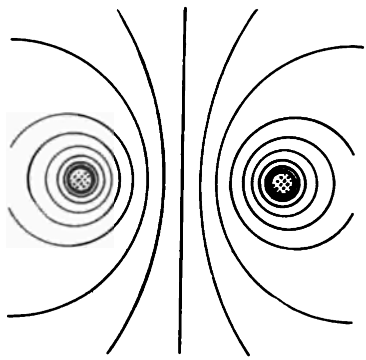

FIG. 2.—Magnetic field of circuit.

conductivity of the conductor in the electric circuits. In empty space $\mu = 1$ and for all non-magnetic materials it is very nearly equal to unity. For magnetic materials the permeability is greatly increased and may reach several thousand for soft iron and steel. The factor 4π comes from the definition of a unit magnetic pole as having one line per cm.[2] on the surface of a sphere of unit radius. The 10^{-1} factor results from the definition of the ampere.

In building up a magnetic field, lines of force cut the conductor and thus produce a counter e.m.f., or inductance

voltage, ${}_Le$, which is equal to the time rate of change of the interlinked magnetic flux.

$$ {}_Le = \frac{d\Phi}{dt} = L\frac{di}{dt} \tag{5} $$

Necessarily an equal opposite voltage must be impressed to force the current through the electric circuit. The product of the voltage and the current represents the power required to generate the field. Hence, the energy stored in a magnetic field by a current, I, in a circuit having an inductance, L, is given by equations (6) and (7).

$$ \int_0^W dw = \int_0^I {}_Leidt = L\int_0^I idi \tag{6} $$

$$ W = \frac{LI^2}{2} \tag{7} $$

The energy is stored magnetically in the electric field surrounding the conductor and is proportional to the square of the current. When the current decreases the energy is returned to the circuit, for if i and therefore ϕ decrease, di/dt and hence ${}_Le$ are negative, which means that the energy is returned to the electric circuit.

The practical unit of inductance, L, is the henry. In any consistent system of units a circuit possesses one unit of inductance, if a unit rate of change of current in the circuit generates or consumes one unit of voltage. If the current changes *at the rate of one ampere per second,* and the voltage generated or consumed is *one volt,* then the inductance is *one henry.*

Dielectric Circuit.—For the dielectric field similar relations exist. All dielectric lines of force are continuous and end on conductors. Ohm's Law may be applied to the dielectric circuit in the same manner as to the magnetic and electric circuits.

$$ \text{Dielectric flux} = \frac{\text{voltage}}{\text{elastance}}; $$

$$ \Psi = \frac{e}{S} = Ce \tag{8} $$

The dielectric flux is directly proportional to the voltage between the conductors and inversely proportional to the elastance of the dielectric circuit. The elastance, S, is the reciprocal of the condensance, C, and varies directly as the length, x, and inversely as the cross section, A, of the dielectric circuit. It corresponds to resistance of the electric circuit and to reluctance of the magnetic circuit.

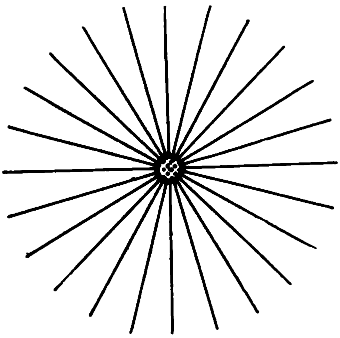

FIG. 3.—Dielectric field of single conductor.

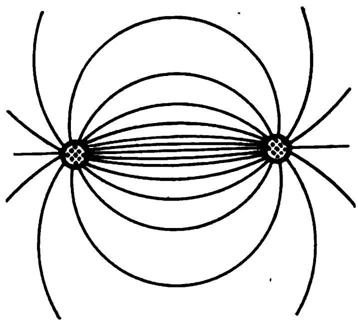

FIG. 4.—Dielectric field of circuit.

$$S = \frac{4\pi x}{\kappa A};\ C = \frac{\kappa A}{4\pi x} \text{ in c.g.s. electrostatic units} \tag{9}$$

$$S = \frac{4\pi v^2 x}{\kappa A};\ C = \frac{\kappa A}{4\pi v^2 x} \text{ in electromagnetic units} \tag{10}$$

$$S = \frac{4\pi v^2 x}{\kappa A 10^9} = 113.1\ \frac{10'' x}{\kappa A} \text{ darafs} \tag{11}$$

$$C = \frac{\kappa A 10^9}{4\pi v^2 x} = 88.42\ \frac{\kappa A}{x} 10^{-15} \text{ farads} \tag{12}$$

$$C = 88.42\ \frac{\kappa A}{x}\ 10^{-9} \text{ microfarads} \tag{13}$$

The permittivity κ is unity for empty space and very nearly equal to unity for air and many other materials. In Table I is given the permittivity constants for the more common dielectrics used in electric apparatus. The constant $v = 3{\cdot}10^{10}$ cm/sec., the velocity of the propagation of an electric field in space (equivalent to the velocity of light), is the ratio of the units used in the electromagnetic

and electrostatic systems. The factor 4π comes from the definition of a unit line of dielectric force.

TABLE I

Material	Permittivity	Material	Permittivity
Air and other gases	1.0	Olive oil	3.0 to 3.2
Alcohol, amyl	15.0	Paper with turpentine	2.4
Alcohol, ethyl	24.3 to 27.4	Paper or jute impregnated	4.3
Alcohol, methyl	32.7	Paraffin	2.3
Asphalt	4.1	Paraffin oil	1.9
Bakelite	6.6 to 16.0	Petroleum	2.0
Benzine	1.9	Porcelain	5.3
Benzol	2.2 to 2.4	Rubber	2.4
Condensite	6.6 to 16.0	Rubber vulcanized	2.5 to 3.5
Glass (easily fusible)	2.0 to 5.0	Shellac	2.7 to 4.1
Glass (difficult to fuse)	5.0 to 10.0	Silk	1.6
Gutta-percha	3.0 to 5.0	Sulphur	4.0
Ice	3.0	Turpentine	2.2
Marble	6.0	Varnish	2.0 to 4.1
Mica	5.0 to 7.0		
Micarta	4.1		

The charging current, ${}_ci$, storing energy in the dielectric circuit is equal to the time rate of change in the dielectric flux.

$$ {}_ci = \frac{d\Psi}{dt} = C\frac{de}{dt} \tag{14} $$

Hence the energy stored in the dielectric field by a voltage, E, in a circuit having a condensance, C, is given by equations (15) and (16):

$$ \int^W dw = \int_o^E {}_ciedt = C\int_o^E ede \tag{15} $$

$$ W = \frac{CE^2}{2} \tag{16} $$

The energy stored dielectrically in the electric field surrounding a conductor is proportional to the square of the voltage. When the voltage decreases the energy is returned to the electric circuit, for if e and therefore ψ

decreases, then de/dt and hence $_ci$ are negative, which means that the energy is returned to the electric circuit. The unit of condensance (capacitance), C, is the farad. In any consistent system of units a circuit possesses one unit of condensance if a unit rate of change of voltage produces (or consumes) one unit of current. If the voltage changes *at the rate of one volt per second* and the current produced (or consumed) is *one ampere*, the condensance

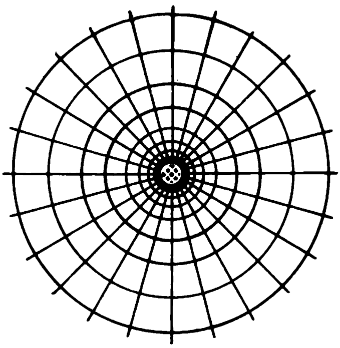

FIG. 5.—Electric field of conductor.

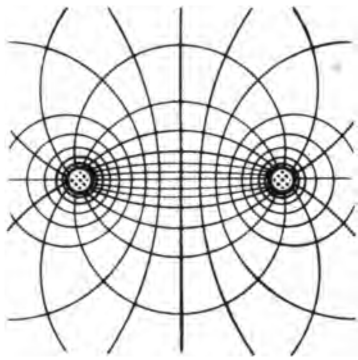

FIG. 6.—Electric field of circuit.

of the circuit is *one farad*. The farad is too large a unit for practical purposes and hence in commercial problems the condensance is usually measured in microfarads.

$$1 \text{ farad} = 10^6 \text{ microfarads} \qquad (17)$$

Electric Circuit.—The electric circuit relates specifically to the conductor carrying the electric current although the term is frequently made to include the dielectric and magnetic fields, since the electric, dielectric and magnetic circuits are interlinked. Under steady or permanent conditions in a direct current system the electric circuit transmits the energy without causing any change in the energy stored magnetically and dielectrically in the space surrounding the electric circuit. In starting the system a transient condition exists until the magnetic and dielectric fields have been supplied with the required amount of energy as determined by the magnitude of the current and voltage and the circuit constants.

If the electric circuit be considered as something separate and apart from the surrounding magnetic and dielectric fields, no storage of energy would be involved and hence no transients could exist, since all the changes would be instantaneous. But the electric circuit is interlinked with the dielectric and magnetic circuits. Changes in the current and voltage in the electric circuit are accompanied by changes in the energy stored in the dielectric and magnetic fields, thus necessitating a readjustment of the energy content in the whole electric system. The transfer of energy requires time and thus the transient period is of definite, although often of extremely short, duration.

The close analogy existing between electric, dielectric and magnetic circuits may be shown to advantage by arranging the corresponding quantities in tabular form as in Table II.

For convenience in solving problems the energy equations are expressed in the units used in commercial work:

Energy in a Magnetic Field:

$$= W(\text{joules}) = \frac{L(\text{henrys})\, i^2\,(\text{amperes})}{2} \tag{18}$$

Energy in a Dielectric Field:

$$= W(\text{joules}) = \frac{C(\text{microfarads})\, e^2\,(\text{volts})}{2 \times 10^6} \tag{19}$$

Energy in a Moving Body:

$$= W(\text{ergs}) = \frac{M(\text{grams})\, v^2\,(\text{meters per sec.})}{2} \tag{20}$$

Energy in a Moving Body:

$$= W(\text{joules}) = \frac{M(\text{kg.})\, v^2\,(\text{meters per sec.})}{2}$$

Energy in a Moving Body:

$$= W(\text{ft.-lb.}) = \frac{M(\text{lb.})\, v^2\,(\text{ft. per sec.})}{2 \times 32.2}$$

$$1 \text{ joule} = 1 \text{ watt-sec.} = 10^7 \text{ ergs} = 0.7376 \text{ ft.-lb.}$$
$$= 0.2389 \text{ g.-cal.} = 0.102 \text{ kg.-m.} \doteq 0.0009480 \text{ B.t.u.} \tag{21}$$

$$1 \text{ ft.-lb.} = 1.356 \text{ joules} = 0.3239 \text{ g.} = 0.1383 \text{ kg.-m.}$$
$$= 0.001285 \text{ B.t.u.} = 0.0003766 \text{ watt-hour} \tag{22}$$

$$1 \text{ B.t.u.} = 1{,}055 \text{ joules} = 778.1 \text{ ft.-lb.} = 252 \text{ g.-cal.}$$
$$= 0.2930 \text{ watt-hour} \tag{23}$$

TABLE II

Electric circuit	Dielectric circuit	Magnetic circuit
Electric current: $i = Ge = \frac{e}{R}$ electric current.	**Dielectric flux (dielectric current):** $\Psi = Ce = \frac{e}{S}$ lines of dielectric force.	**Magnetic flux (magnetic current):** $\phi = Li\,10^8$ lines of magnetic force.
Electromotive force, voltage: e = volts.	**Electromotive force:** e = volts.	**Magnetomotive force:** $\mathfrak{F} = 4\pi ni$ ampere-turns. gilberts.
Conductance: $G = \frac{i}{e}$ mhos.	**Condensance, capacitance, permittance or capacity** $C = \frac{\Psi}{e}$ farads.	**Inductance:** $L = \frac{n\phi}{\mathfrak{F}}10^{-8} = \frac{\phi}{i}10^{-8}$ henrys
Resistance: $R = \frac{e}{i}$ ohms.	**Elastance:** $S = \frac{i}{C} = \frac{e}{\Psi}$ darafs.	**Reluctance:** $R = \frac{\mathfrak{F}}{\phi}$ oersteds.
Electric power: $p = Ri^2 = Ge^2 = ie$ watts.	**Dielectric energy:** $w = \frac{Ce^2}{2} = \frac{\Psi e}{2}$ joules.	**Magnetic energy:** $w = \frac{Li^2}{2} = \frac{\phi i}{2}10^{-8}$ joules.
Electric-current density: $I = \frac{i}{A} = \gamma G$ amp. per cm.^2	**Dielectric-flux density:** $D = \frac{\varphi}{A} = \kappa K$ lines per cm.^2	**Magnetic-flux density:** $B = \frac{\phi}{A} = \mu H$ lines per cm.^2
Electric gradient: $G' = \frac{e}{l}$ volts per cm.	**Dielectric gradient** $G' = \frac{e}{l}$ volts per cm.	**Magnetic gradient:** $f = \frac{\mathfrak{F}}{l}$ amp.-turns per cm.
Conductivity: $\gamma = \frac{I}{G}$ mho.-cm.^3	**Condensivity, permittivity or specific capacity:** $k = \frac{D}{K}$	**Permeability:** $\mu = \frac{B}{H}$
Resistivity: $\rho = \frac{1}{\gamma} = \frac{G}{I}$ ohm-cm.^3	**Elastivity:** $\frac{1}{k} = \frac{K}{D}$	**Reluctivity:** $v = \frac{f}{B}$
Specific electric power: $p = \rho I^2 = \gamma G^2 = GI$ watts per cm.^3	**Specific dielectric energy:** $w = \frac{kG'^2}{4\pi v^2} = \frac{G'D}{2}$ joules per cm.^3	**Specific magnetic energy-** $w = \frac{0.4\pi\mu f^2}{2} = \frac{fB}{2}10^{-8}$ joules per cm.^3
	Condensance, permittance, capacitance current: $_ci = \frac{d\Psi}{dt} = C\frac{de}{dt}$ amperes.	**Inductance voltage:** $e = \frac{d\phi}{dt}10^{-8} = L\frac{di}{dt}$ volts.
	Dielectric-field intensity: $K = \frac{G'}{4\pi v^2}$ lines of dielectric force per cm.^2	**Magnetic-field intensity:** $H = 4\pi f 10^{-1}$ lines of magnetic force per cm.^2

CHAPTER II

OSCILLOGRAPHS

The oscillograph is the most important apparatus for obtaining quantitative data on electric transient phenomena. To gain clear concepts of the relative magnitude of the physical quantities involved it is highly desirable for

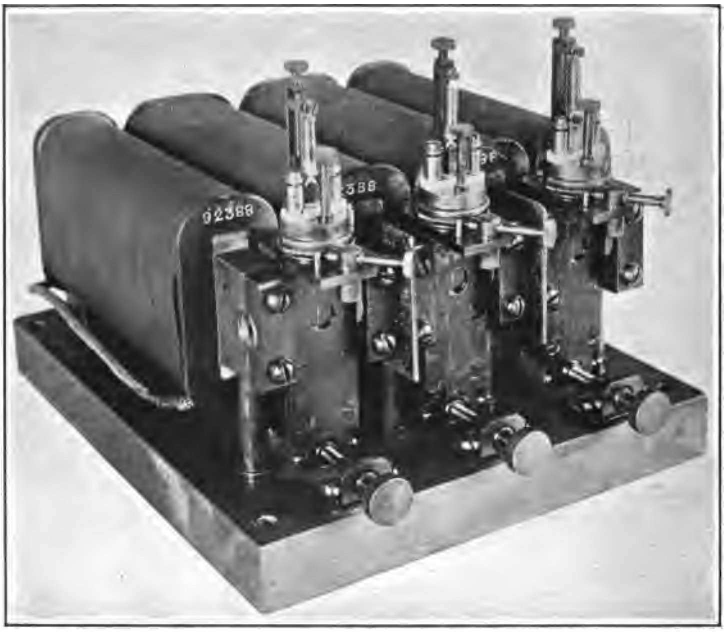

FIG. 7.—Magnetic field and vibrating elements.

the student to take oscillograms of a number of typical transients. For this purpose an oscillograph with a photographic attachment is necessary.

While several types of oscillographs are in commercial use all operate on the same basic principle. The essential

element of the oscillograph is the galvanometer, an insulated loop of wire, placed in a magnetic field, through which the electric current flows. The direction and magnitude of the currents cause a proportional turning movement of a small mirror attached to both sides of the loop. The deflection of a beam of light thrown on the mirror indicates the angular position of the mirror and hence the magnitude and direction of the current flowing through the loop.

Three Element Oscillographs.—The three element, portable type oscillograph manufactured by the General Electric Co. is shown in Figs. 7 to 13. The arrangement of the

Fig. 8.—Vibrating element.

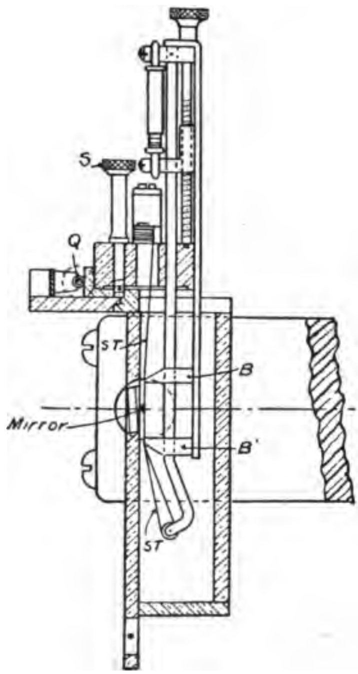

Fig. 9.—Cross section of vibrating element.

electromagnetic field and the three vibrating elements is shown in Fig. 7. One of the vibrating elements removed from its magnetic field is shown in Fig. 8 and its vertical cross-section in Fig. 9. The three vibrators are indepen-

dent units and insulated so as to carry three separate electric currents. The vibrating strips and mirrors are of silver. The vibrating element can be turned around a vertical axis, passing through the center of the mirror, by the screw Q. The containing cell for the whole vibrating element is also movable around a horizontal axis, passing through the center of the mirror, by means of the screw S. Hence the beam of light reflected from the vibrating mirror may be directed to any desired spot and so adjusted as to pass through the cylindrical lens to the slit in front of the rotating photographic film.

In Figs. 7 and 8, the letters TT' mark the terminals of the vibrating strips marked ST in Fig. 9. The mirror with the vibrating portion of the loop lies between the supports BB'. The size of the mirror is about 20 by 10 mils and the vibrating element has a natural period of approxi-

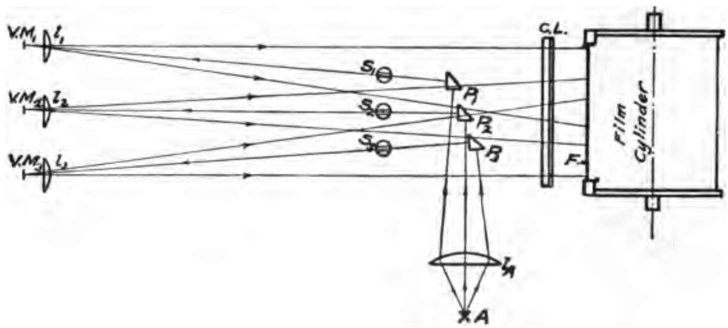

Fig. 10.—Optical train—horizontal projection.

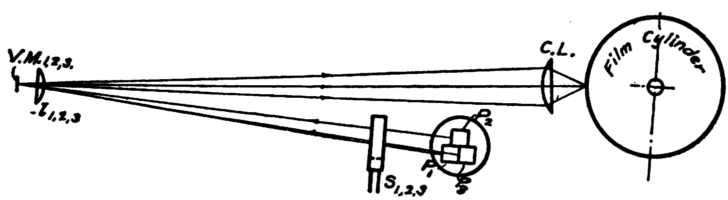

Fig. 11.—Optical train—vertical projection.

mately one five-thousandth of a second (0.0002 sec.). By immersing the vibrating element in oil the instrument is made dead-beat.

The horizontal projection of the optical train for photographic work is shown in Fig. 10 and a vertical projection in Fig. 11. The arc lamp is at A and the arrows indicate the directions of the beams of light. P_1, P_2, P_3 are right-angled prism mirrors; S_1, S_2, S_3, adjustable slits; l_1, l_2, l_3 condensing lenses; VM_1, VM_2, VM_3 the vibrating mirrors; CL a cylindrical lens for bringing the light beams to a sharp focus on the photographic film on the surface of the revolving cylinder in the film holder.

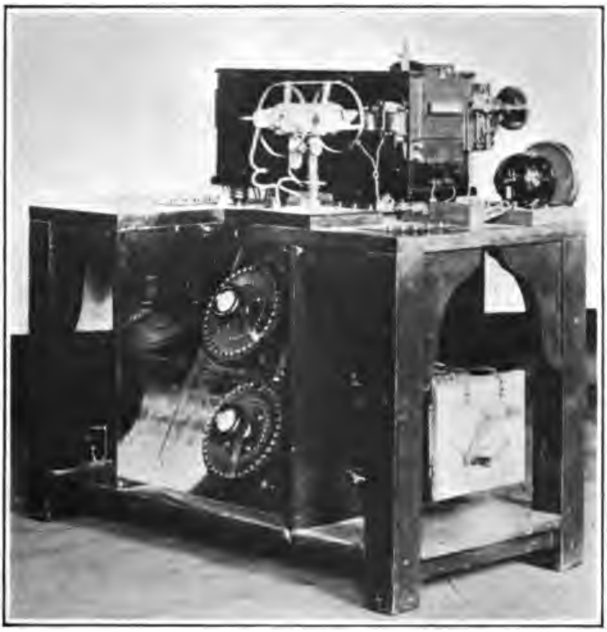

FIG. 12.—Oscillograph on operating stand. (*Gen. Elec. Co.*)

In Fig. 12 the oscillograph is shown mounted on a conveniently arranged operating stand. The positions of the arc lamp, film motor, film holder, controlling rheostats, time wave oscillator and other accessory appliances for recording electric transients are clearly indicated. The corresponding wiring diagram, with quantitative circuit data, is shown in Fig. 13.

The three-element, portable oscillograph of compact design, manufactured by the Westinghouse Elec. & Mfg. Co., is shown in Figs. 14 to 16. The photographic film drum and driving pulley with rheostats, switches, etc., are shown on the right side of Fig. 14*a*, while on the left are the

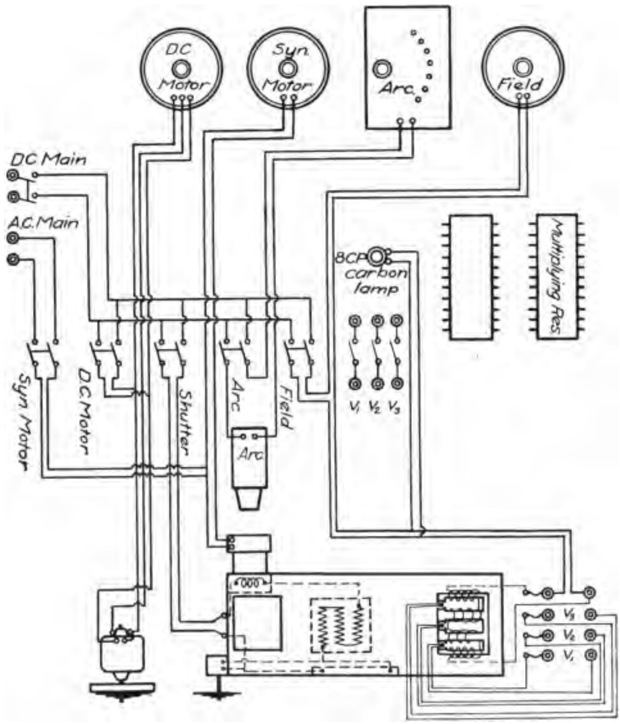

FIG. 13.—Wiring diagram—three element oscillograph. (*Gen. Elec. Co.*)

three sets of dial resistances, one for each vibrating element, with switches, binding posts and fuses. In Fig. 14*b* is shown the driving motor with control apparatus for operating the film holder at several speeds. Light for making the photographic record is obtained from a low voltage incandescent lamp of special design. For high speed records an arc lamp is used, in place of the incandescent lamp, to gain the greatest possible light intensity.

The galvanometer, with one of the three elements re-

FIG. 14a.—Front and resistance panel side of portable oscillograph. (*Westinghouse Elec. & Mfg. Co.*)

FIG. 14b.—Front view of portable oscillograph coupled to motor. (*Westinghouse Elec. & Mfg. Co.*)

moved, is shown in Fig. 15. The moving element consists of a single turn or oblong loop of wire forming two parallel conductors. A tiny mirror is attached to both conductors and placed in a strong magnetic field. Hence when a current passes down one conductor and up the other, one tends to move forward and the other backward. The mirror bridging these conductors is given an angular deflection proportional to the current.

FIG. 15.—Three element galvanometer. (*Westinghouse Elec. & Mfg. Co.*)

The design of the electromagnetic field circuit is unique. A direct current passing through a single coil sets up a magnetic flux which passes through the three vibrating elements in series. To insulate the elements from each other and from the main magnetic core and yokes four insulating gaps are used, thus placing seven air gaps in series in the path of the magnetic flux. The three gaps in the galvanometer elements are $\frac{1}{32}$ in. long, giving sufficient space for the vibrators and producing uniform distribution of the magnetic flux. The four insulating gaps are $\frac{1}{16}$ in. long but of large cross-sectional area so as to give comparatively low reluctance in the magnetic circuit. The field excitation requires 6 volts, direct current.

A view of the trip magnet and shutter release mechanism

is shown in Fig. 16, in which the trip magnet holds the long shutter finger so that the short finger does not quite touch the shutter tripping arm. The shutter is a tube with two opposite longitudinal slots. The tube rotates and when the slots are in a horizontal plane the beams of light, reflected from the tiny mirrors of the galvanometer vibrators, pass through the cylindrical condensing lens and are focused on the revolving photographic film. This occurs between the time the short finger falls from the shutter

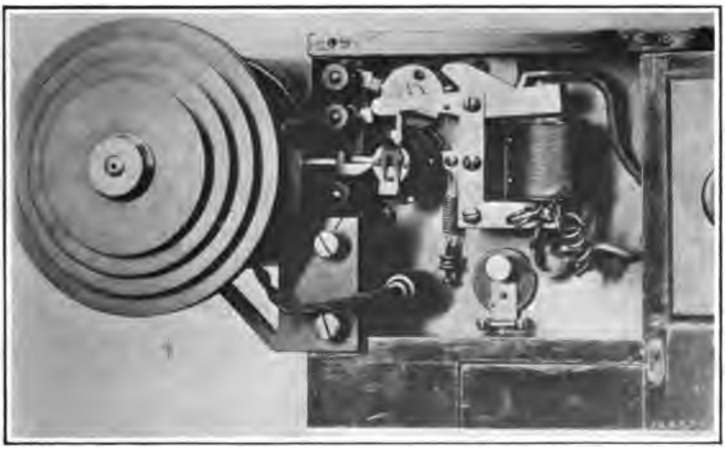

FIG. 16.—Trip magnet and shutter release mechanism. (*Westinghouse Elec. & Mfg. Co.*)

tripping arm and the time the variable finger falls from the arm one revolution later. The shutter is actuated by the spiral spring seen just beyond the finger hub. A pin on the shutter shaft strikes an arm on the lamp extinguishing switch. On the hub are attached laminated copper strips which complete the lamp circuit when the shutter is set and which break the circuit when the shutter snaps closed. The tripping device can be adjusted so as to start exposures at any desired part of the film.

Timing-waves from the Oscillator Alternator.—The time factor is of special importance in electric transient phenomena and some means for recording the time elapsed

is necessary. In taking oscillograms in which the transient current or voltage recorded does not give directly an indication of the time consumed it is customary to impress an alternating current timing wave of known frequency on one of the vibrators.

Current for the timing wave may be taken directly from any available power circuit, but the frequencies of commercial systems are to some extent variable and the indicating frequency meters may not be sufficiently accu-

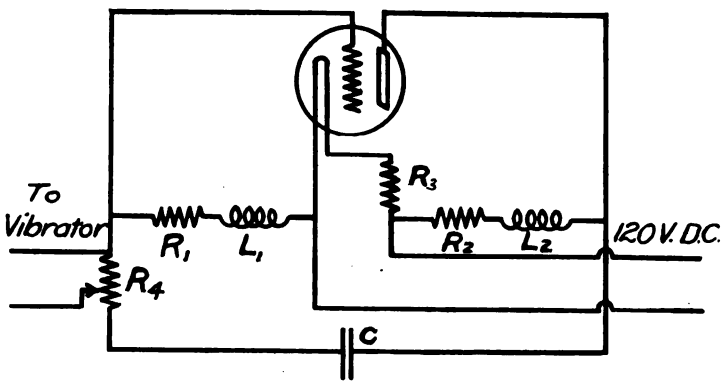

FIG. 17.—Oscillator alternator circuit diagram.

rate for this purpose. A convenient source of supply for timing wave current of constant frequency is found in the oscillator generator. The circuit diagram of a simple portable form used in the electrical engineering laboratories of the University of Washington is shown in Fig. 17. The alternator consists of an audion tube connected to condensance, resistance and inductance, as shown in the circuit diagram, of the following quantitative values:

$L_1 = 0.756$ henrys	$R_1 = 99$ ohms
$L_2 = 0.756$ henrys	$R_2 = 99$ ohms
$L(\text{total}) = 2.38$ henrys	$R_3 = 96$ ohms
$C = 1.06$ microfarads	$R_4 = 50$ ohms

The impressed d.c. voltage was 110 volts but other values may be used by adjusting the resistance, R_3. The amplitude of the timing wave may be varied by means of the

FIG. 18.—Discharging a condenser through a resistance. See Chap. III.

resistance, R_4. The alternating current produced by the oscillator is of simple sine wave form and has a constant frequency of 100 cycles per second.

Oscillograms.—Great care must be taken in making the adjustments on the oscillograph in order to produce good oscillograms. The speed of the revolving drum carrying the sensitized film and the amplitude of the galvano-

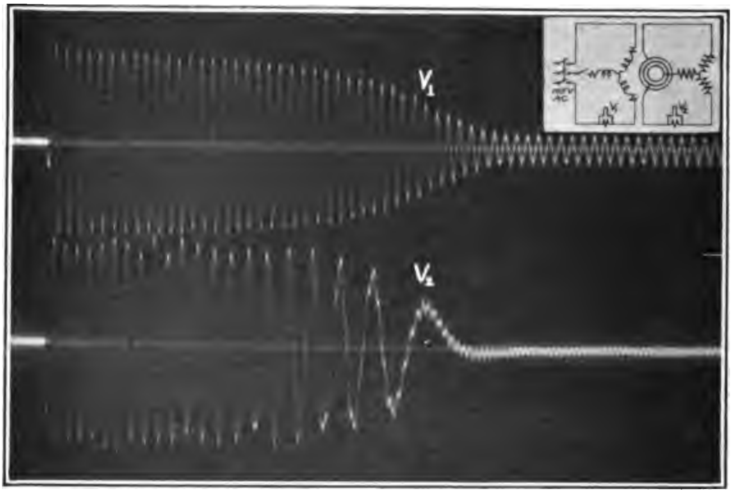

FIG. 19.—Starting transient of induction motor with secondary resistance in circuit. See Fig. 167, Chap. IX.

meter mirror vibrations must be adjusted to meet the conditions imposed by the transient under investigation. Thus the relative drum speed for the oscillograms shown in Figs. 18, 19, and 20 was as 4 : 1 : 29, and the amplitude adjusted in each case so as to use the film area to good advantage.

The time lag of tripping devices and shutter operating mechanism must be determined so as to expose the film at the instant the transient occurs. The optical train must be adjusted so as to give a spot of light sharply focused on the sensitized film.

Instructions for developing the films and for printing the oscillograms are given in the Appendix. A circuit diagram

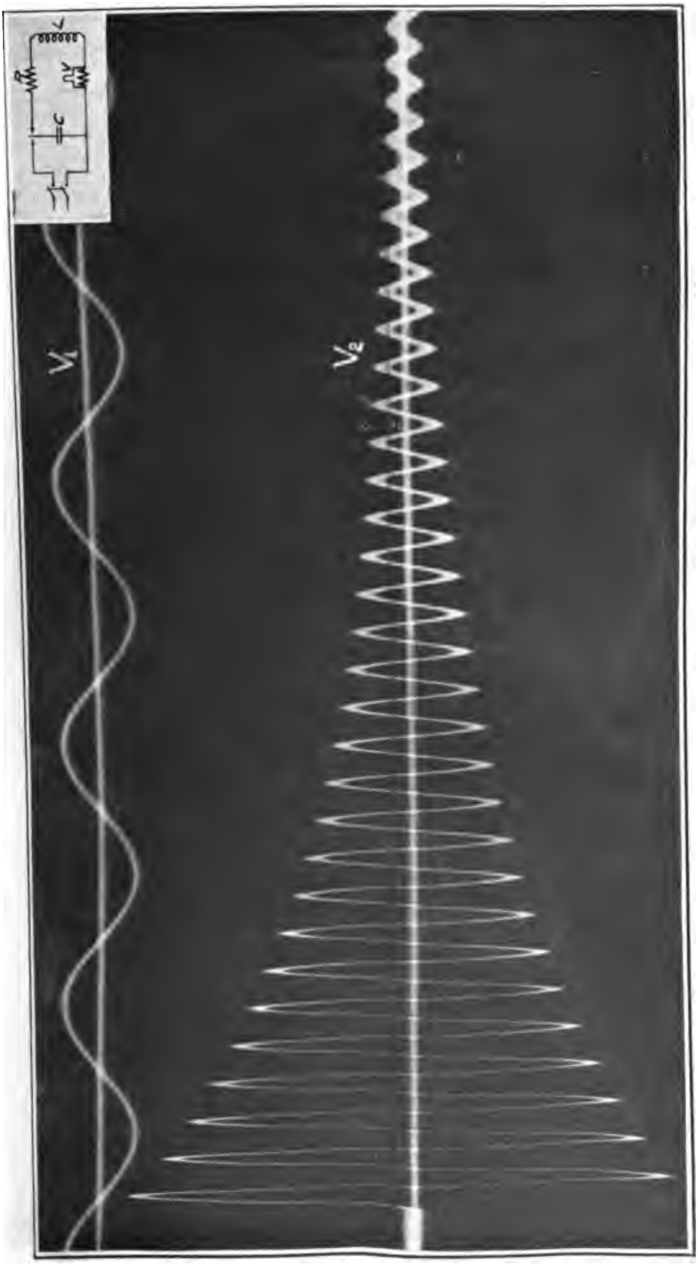

FIG. 20.—Double energy transient. See Chap. V.

with quantitative data should be attached to each film. It is important to show the circuit position of each vibrator

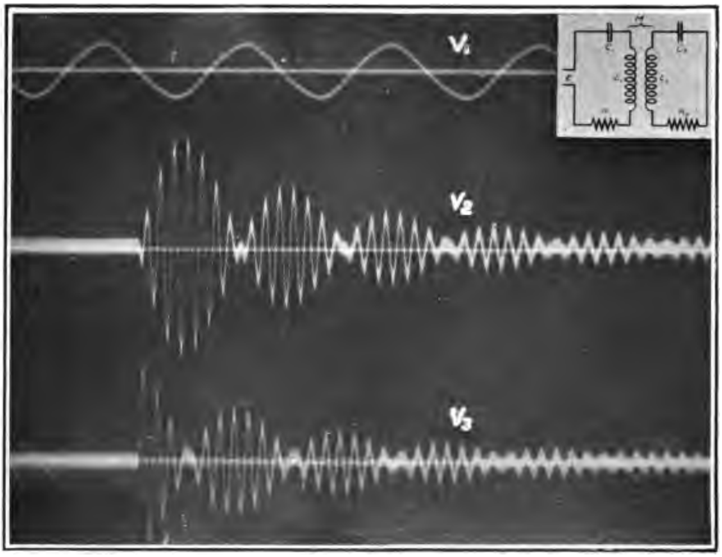

FIG. 21.—Transfer of oscillating energy in inductively coupled circuits. See Chap. VIII.

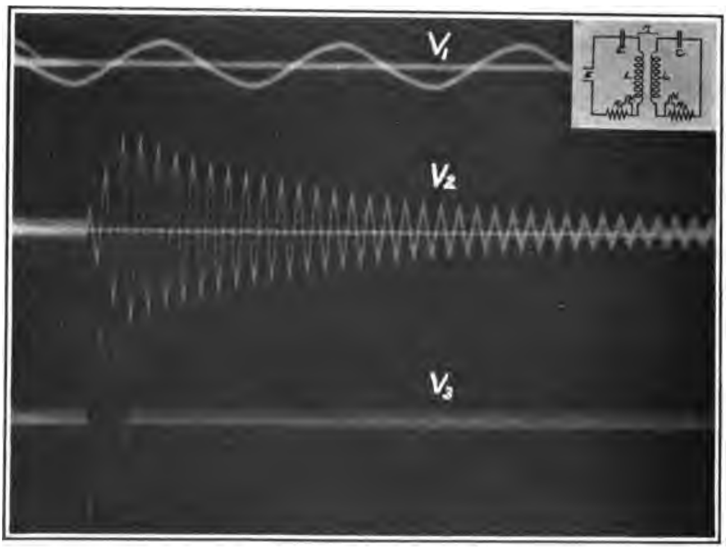

FIG. 22.—Same circuit as in Fig. 21. Primary opened at the instant all the oscillating energy was in the secondary circuit.

so that the record will indicate precisely where the transient appearing on the film was taken.

Problems and Experiments

1. Examine the oscillograph with care; trace all the circuits; operate the arc lamp; adjust the optical train until the mirror on each vibrator throws a spot of light through the slit and this is sharply focused on the ground glass screen. Arrange a circuit with variable inductance and condensance as indicated in Fig. 23. Connect vibrator V_1 by means of a shunt, S, so

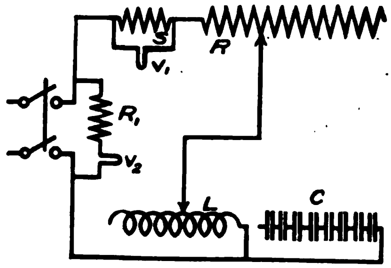

FIG. 23.

as to indicate the current wave and vibrator V_2 with a resistance, R_1, in series to show the voltage wave. By means of the small synchronous motor operate the large oscillating mirror throwing the beams of light on the mica screen on top of the oscillograph. By varying the resistance, inductance and condensance in the circuit the time phase relations of the voltage and current may be changed from lag to lead.

2. Reproduce, as nearly as available equipment will permit, the oscillogram in Fig. 18.

CHAPTER III

SINGLE ENERGY TRANSIENTS. DIRECT CURRENTS

Transient electric phenomena are produced by changes in the magnitude, distribution and form of the energy stored in electric systems. The simplest types of electric transients are found in electric circuits having only one kind of energy storage—that is, either the magnetic or the dielectric field, but not both. A condenser discharging through a non-inductive resistance, as illustrated by the oscillogram in Fig. 24, gives electric transients of the simplest type. Since the resistance in the circuit is constant the current is at all instants directly proportional to the voltage across the terminals of the condenser. The curve on the oscillogram can therefore be used as representing either the current-time or the voltage-time relation as indicated by the two scales in the figure.

The Exponential Law.—The energy stored in the condenser is at any instant equal to $Ce^2/2$. The rate of discharge is ei, which must be equal to the Ri^2 rate of energy dissipation into heat in the resistance. *The rate of energy discharge is therefore at any instant proportional to the energy stored in the condenser and the rate of change in the current is at any instant directly proportional to the magnitude of the current.*

Let i and i' represent the currents at any two points on the current-time curve of the oscillogram, in Fig. 24. Then:

$$\frac{di}{dt} : \frac{di'}{dt} : : i : i' \tag{24}$$

Let the line OP be drawn from starting point O perpendicular to the X axis. Let the line OQ be drawn tangent to the curve at O and intersecting the X axis at Q. The time represented by the distance PQ is called the time constant,

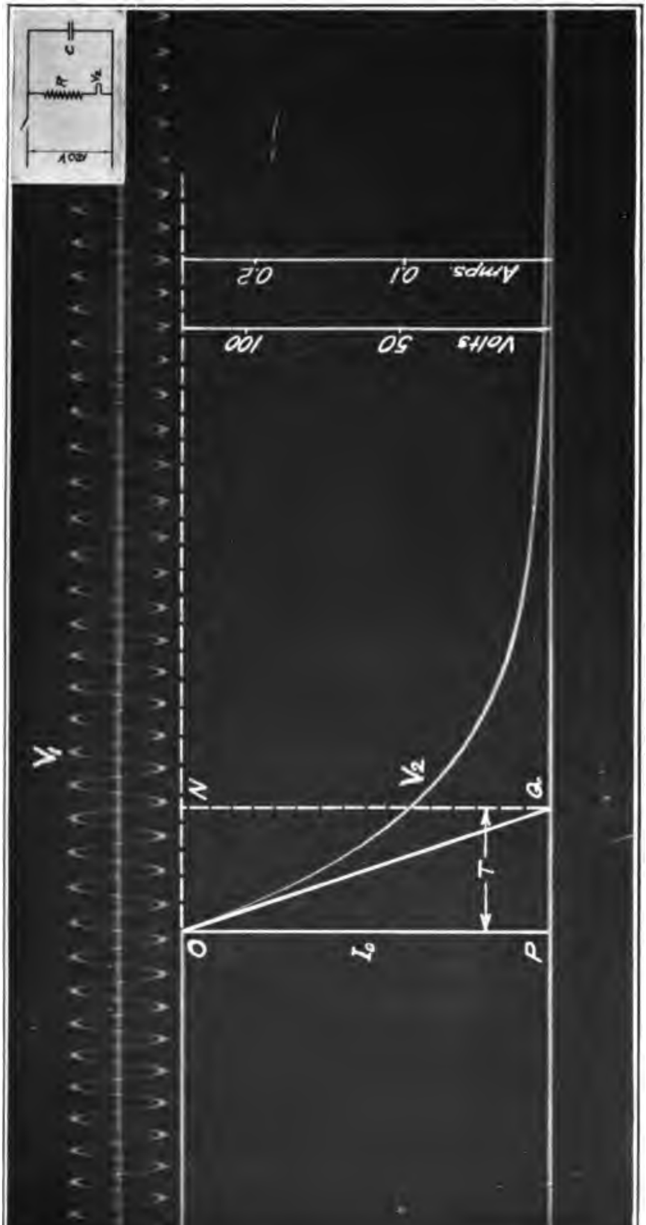

FIG. 24.—Condenser discharging through a constant resistance. E = 120 volts. I = 0.24 amps.; R = 500 ohms, C = 72 m.f.; T = 0.036 seconds.

T, of the circuit. Since i' may be any point on the curve, let it be taken at the starting point, O; then

$$i' = I_0 \text{ and } \frac{di'}{dt} = -\frac{I_0}{T} \tag{25}$$

From (24) and (25)

$$\frac{di}{dt} : -\frac{I_0}{T} :: i : I_0 \tag{26}$$

Separating the variables and taking the limits of integration from the starting point, O, to any point (i, t) on the curve:

$$\int_{I_0}^{i} \frac{di}{i} = -\frac{I}{T}\int_{0}^{t} dt \tag{27}$$

$$\log_\epsilon \frac{i}{I_0} = -\frac{t}{T} \tag{28}$$

$$i = I_0 \epsilon^{-\frac{t}{T}} \tag{29}$$

Similarly, for the corresponding voltage-time curve:

$$e = E_0 \epsilon^{-\frac{t}{T}} \tag{30}$$

Equations (29) and (30) show that *the fundamental relations in simple electric transients are expressed by the exponential equation.* The minus sign is used as di/dt is negative. The exponential curve represented by equations (29) and (30) is as fundamental in the study of electric transients as the sine wave in alternating currents.

If the energy stored in a magnetic field is released by short circuiting through a resistance and dissipated into heat, the same relations exist. Oscillograms of the current-time or voltage-time curves similar to Fig. 24, may be obtained by discharging a magnetic field through a resistance, Fig. 27. Likewise, the electric transients existing while a condenser is charged, or while a magnetic field is established, obey the exponential law. In Fig. 25 is shown the oscillogram of a current-time transient obtained while establishing a magnetic field in the circuit shown in the diagram. Let the line OP be drawn through the starting point O at right angles to the X axis. Let PS be drawn

parallel to the X axis and be an asymptote to the current-time curve. Let the line OQ be drawn tangent to the curve

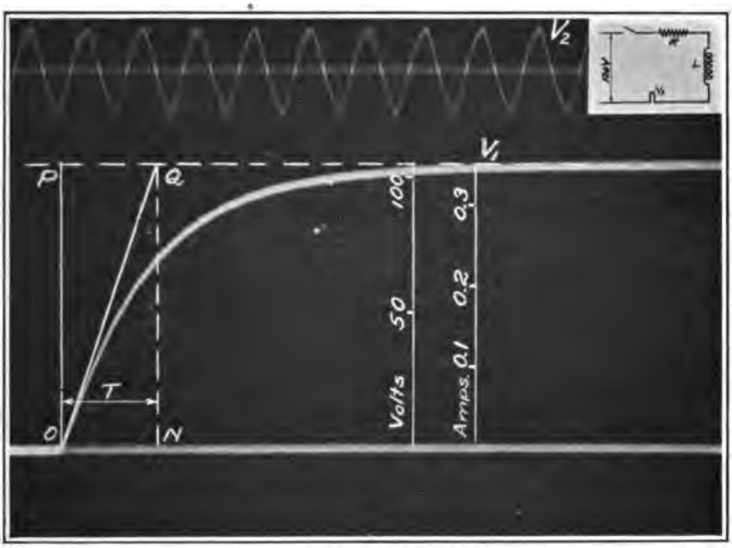

FIG. 25.—Forming a magnetic field.
$E = 109$ volts; $I = 0.36$ amps.; R $= 303$ ohms; $L = 5.1$ henrys; $T = 0.017$ seconds.

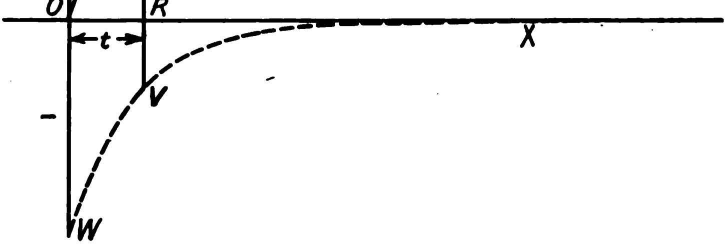

FIG. 26.—Showing permanent, transient and instantaneous values for oscillogram in Fig. 25.

at O and intersecting the line PS at Q. The line OP represents the value of the permanent current I which is equal

to E/R. The time measured by the line PQ is the time constant, T, of the circuit. The rate of storing the magnetic energy at any instant is proportional to the remaining magnetic storage facilities in the circuit under the given conditions. Therefore the rate of change in the current is at any point on the curve proportional to $I - i$, and equation (31) is derived in the same manner as equations (24) and (26)

$$\frac{d(I-i)}{dt} : -\frac{I}{T} :: I - i : I \tag{31}$$

$$\int_o^i \frac{d(i-I)}{i-I} = -\frac{I}{T}\int_o^t dt \tag{32}$$

$$\log_\epsilon\left(\frac{i-I}{(-I)}\right) = -\frac{t}{T} \tag{33}$$

$$i = I - I\epsilon^{-\frac{t}{T}} \tag{34}$$

The transient which by definition represents the change from one permanent condition to another is in equation (34) represented by the factor $-I\epsilon^{-\frac{t}{T}}$. Before the circuit was closed the value of the current was zero, while the final permanent value is I.

In Fig. 26 the permanent or final value of the current is represented by OP, the distance of the line PS from the X axis. The transient values are given by the ordinates to the broken curve WVX, while the instantaneous current which must at any instant be equal to the algebraic sum of the permanent and transient values is given by the ordinates to the curve OYS, which is the curve photographed on the oscillogram in Fig. 25. It is important to note that the photographic record of the actual instantaneous values gives at each point the resultant or the algebraic sum of the corresponding permanent and transient ordinates. Thus for any time, t:

$$RY = RU + (-RV) \tag{35}$$

The Time Constant.—From the starting point O of the current-time curve in the oscillogram, Fig. 27, which shows the discharge of a magnetic field through a resistance, the line OP is drawn at right angles to the X axis; the line OQ tan-

gent to the curve at the point O and intersecting the X axis at Q; the line QN perpendicular and ON parallel to the X axis.

From the principle of the conservation of energy, the energy stored in the magnetic field must be equal to the amount dissipated as heat in the resistance of the circuit when the field is discharged.

$$\frac{LI_0^2}{2} = R\int_0^\infty i^2dt = RI_0^2\int_0^\infty \epsilon^{-\frac{2t}{T}}dt = \frac{RI_0^2T}{2} \tag{36}$$

$$T = \frac{L}{R} \tag{37}$$

In circuits having resistance and inductance in series, as in Fig. 27, the time constant is equal to the inductance divided by the resistance.

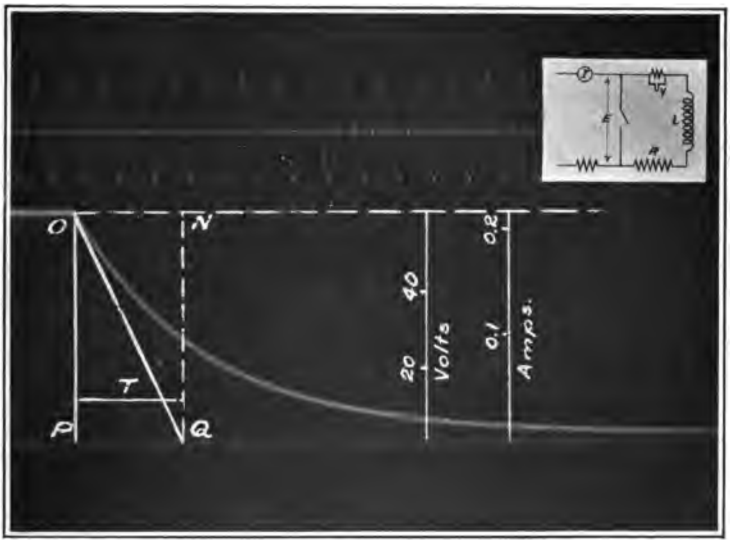

FIG. 27.—Discharge of a magnetic field through a constant resistance. E = 60 volts; I = 0.21 amps.; R = 28.6 ohms; L = 0.89 henrys; T = 0.031 seconds.

In a similar manner the expression for the time constant, T, in terms of the circuit constants may be found for circuits having condensance and conductance, Fig. 24. The energy stored in the condenser when the discharge starts must be equal to the energy expended as heat in the Ri^2 losses.

$$\frac{CE_0^2}{2} = R\int_0^\infty i^2dt = RI_0^2\int_0^\infty \epsilon^{-\frac{2t}{T}}dt = \frac{RI_0^2T}{2} \tag{38}$$

Hence,

$$T = CR = \frac{C}{G} \tag{39}$$

In circuits having resistance and condensance in series, as in Fig. 24, the time constant is equal to the condensance divided by the conductance.

Equations (37) and (39) are of fundamental importance in the study of transient phenomena. The exponential equation for the transients in Figs. 24 and 26 may be rewritten using the value of T as given in (37) and (39) and the data in the circuit diagrams.

Fig. 24, equations (29), (39)

$$i = I_0\epsilon^{-\frac{t}{T}} = I_0\epsilon^{-\frac{C}{G}t} = 4.18\epsilon^{-38.5t}\text{amperes} \tag{40}$$

Fig. 24, equations (30), (39)

$$e = E_0\epsilon^{-\frac{t}{T}} = E_0\epsilon^{\frac{C}{G}t} = 120.6\ \epsilon^{-38.5t}\text{volts} \tag{41}$$

Fig. 25, equations (34), (37)

$$i = I - I\epsilon^{-\frac{t}{T}} = I - I\epsilon^{-\frac{R}{L}t} = 0.36 - 0.36\epsilon^{-58.7t}\text{amperes} \tag{42}$$

The reciprocals of the time constants appearing in the exponential equations as R/L and G/C or such combinations of circuit constants as the complexity of the system may require, are often called the *dissipation constants* or the *attenuation constants* of the circuit.

The expressions for the time constants in equations (37) and (39) may be derived from the current-time and voltage-time curves instead of basing the equations directly on the principle of the conservation of energy. In Fig. 24 the quantity of electricity (coulombs) in the condenser when starting the transient must be equal to the total amount expended when the condenser is discharged, as represented by the area between the current-time curve and the X axis.

$$CE_0 = \int_0^\infty idt = I_0\int_0^\infty \epsilon^{-\frac{t}{T}}\ dt = I_0T \tag{43}$$

Hence,

$$T = C\frac{E_0}{I_0} = CR = \frac{C}{G} = 0.026 \text{ seconds} \tag{44}$$

Similarly, in Fig. 27, the magnetic flux in the field when starting the transient may be equated to the total number of lines of force cutting the circuit when all the magnetic energy in the field changes into heat in the resistance.

$$LI_0 = \int_o^\infty edt = R\int_o^\infty idt = RI_0\int_o^\infty \epsilon^{-\frac{t}{T}}dt = RI_0T \tag{50}$$

Hence,

$$T = \frac{L}{R} = 0.31 \text{ seconds} \tag{51}$$

If the initial rate of discharge in Fig. 24 be continued unchanged, the current-time curve would coincide with the line OQ and the condenser would be completely discharged in the time represented by PQ or T. Hence the area of the rectangle $OPQN$ must be equal to the area between the current-time curve and the X axis.

Similarly, in Fig. 27, if the initial rate of discharge continued unchanged, the current-time curve would coincide with the line OQ and all the energy stored in the magnetic field would appear as heat in the resistance in the time represented by PQ or T. Hence the area of the rectangle $OPQN$ must be equal to the area between the current-time curve and the X axis.

Expressions for the transient current and voltage as given in (40), (41) and (42) are derived without using the time constant term. The customary differential equations giving the basic relations, with expressions for the transient currents, are given in (52) to (55).

For circuits having resistance and condensance, Fig. 24, while the condenser is discharged through a constant resistance:

$$Ri + \int\frac{idt}{C} = 0; \text{ hence } i = I_0\epsilon^{-\frac{t}{RC}} = I_0\epsilon^{\frac{-G}{C}t} \tag{52}$$

For circuits having resistance and condensance, similar to Fig. 24, the transient current while charging from O to the voltage E is the same as for discharging through the resistance.

$$Ri + \int \frac{idt}{C} = E; \text{ hence } i = I_0\epsilon^{-\frac{1}{RC}t} \tag{53}$$

For circuits having resistance and inductance, Fig. 19, while the magnetic field is formed:

$$Ri + L\frac{di}{dt} = E; \text{ hence, } i = \frac{E}{R} - \frac{E}{R}\epsilon^{-\frac{R}{L}t} = I - I\epsilon^{-\frac{R}{L}t} \tag{54}$$

For circuits having resistance and inductance, Fig. 21, and a magnetic field supplied by a current I_0, while the field discharges through a short circuit:

$$Ri + L\frac{di}{dt} = 0; \text{ hence, } i = I_0\epsilon^{-\frac{R}{L}t} \tag{55}$$

The Exponential Curve.—Oscillograms of simple electric transients give a photographic record of the current-time factors. The amplitude of the curve varies directly with the magnitude of the current passing through the vibrator and the strength of the magnetic field in which the vibrator moves. The length of film used for any given unit of time depends on the speed of the revolving drum carrying the film. It is evident that both the amplitude of the mirror vibrations and the speed of the film may be adjusted independently of the circuit in which the transient occurs. By examining exponential equations representing simple electric transients it is apparent that if the value of the time constant, T, be used as the unit of length on the X axis and the initial value of the variable as the unit of measure for the ordinates, then all exponential transients will have the same shape and may be represented by the numerical values of the exponential equation, $y = \epsilon^{-x}$. The same space unit need not be used on both axes to represent the unit values of current and time, but the scale may be selected so as to secure a convenient shape for the available

space. In Fig. 28 is shown a current-time curve in which the unit representing the initial value of the transient current is five cm., while the unit used on the X axis, that is, for the time constant of the circuit, is one cm.

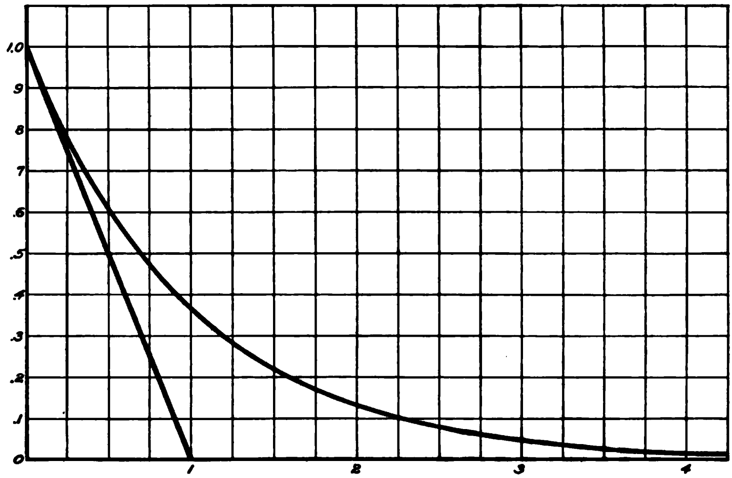

FIG. 28.—The exponential curve. Current-time transient.

By using the initial value of the transient as the unit of ordinates and the time constant of the circuit as the unit

TABLE III

$y = \epsilon^x;\ \epsilon = 2.71828$

x	y	x	y
0.00	1.000	1.2	0.301
0.05	0.951	1.4	0.247
0.10	0.905	1.6	0.202
0.15	0.860	1.8	0.165
0.20	0.819	2.0	0.135
0.30	0.741	2.5	0.082
0.40	0.670	3.0	0.050
0.50	0.607	3.5	0.030
0.60	0.549	4.0	0.018
0.70	0.497	4.5	0.011
0.80	0.449	5.0	0.007
0.90	0.407	6.0	0.002
1.00	0.368	7.0	0.001

of abscissae, all exponential transients are of the same shape and if plotted to the same scale would be identical with the curve in Fig. 28. The numerical relations between y and x in the exponential equation $y = \epsilon^{-x}$ are given in Table III. While the plotting of transients may be facilitated by the selection of the above units, the actual initial values of the transient quantity, expressed in amperes or volts, may be of any magnitude as determined by the circuit conditions.

Initial Transient Values.—In simple electric transients the initial value of the variable quantity depends on both the permanent value and on the relative magnitude of the circuit constants. Thus equations (33) and (42) show that the time constant of a magnetic field depends on the inductance and resistance in the circuit. If the energy stored in the magnetic field be discharged by short circuiting the terminals of the field, the initial value of the transient voltage will be equal in magnitude but opposite in direction to the previously permanent value. But if the discharge be made through an additional resistance, R_2, the initial voltage transient will be greater in magnitude in the ratio of $R_1 + R_2 : R_1$, when R_1 represents the resistance of the field winding. The time constant of the circuit in which the transients appear would be,

$T = \dfrac{L}{R_1}$ when the field is short circuited,

and $T_1 = \dfrac{L}{R_1 + R_2}$ when the additional resistance R_2 is inserted in the discharging circuit. With the same amount of energy stored in the magnetic field, the products of the initial value and the corresponding time constants must be equal.

$$E_0T = E_0'T' \tag{56}$$

Hence,

$$E_0:E_0'::R_1:R_1 + R_2 \tag{57}$$

$$E_0' = E_0\frac{R_1 + R_2}{R_1} \tag{58}$$

The initial induced discharge voltage is therefore greater than the permanent impressed voltage in the proportion of the resistances in circuit for the two cases. In the voltage-time curve, Fig. 29, the initial discharge voltage, E_0'', is that part of the induced voltage, E_0', due to the Ri^2 drop.

$$E''_0 = E'_0\frac{R_1}{R_1 + R_2} = E_0\frac{R_2}{R_1} \qquad (59)$$

This relation is of great importance in the design and operation of electrical machinery. In breaking electric circuits, as induction coils, motor and generator fields, transmission

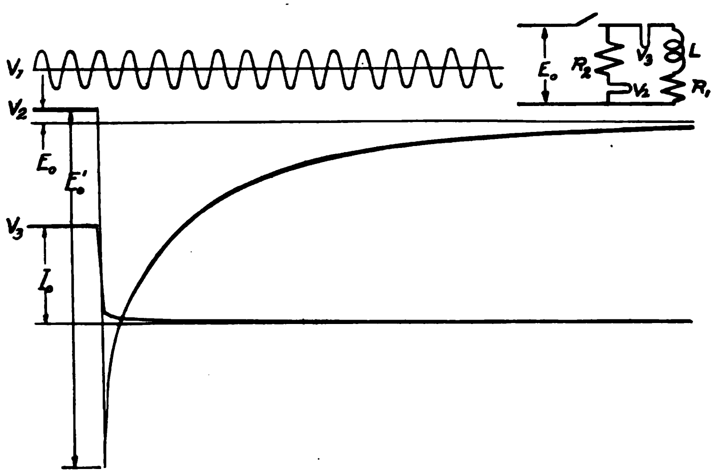

Fig. 29.—Magnetic field discharging through additional resistance, R_2.

lines, etc., in which energy is stored magnetically, the air or oil gap in the switch introduces a rapidly increasing resistance. The faster the contact points of the switch or circuit breaker separate, the more rapidly the resistance is inserted and the higher the induced voltage.

In Fig. 30 is shown the voltage-time and current-time oscillograms for breaking the field circuit of a direct-current motor. In opening the switch an arc is formed by which a resistance of rapidly increasing magnitude is

introduced into the circuit. The oscillogram shows that in about $1/15$ of a second the induced voltage increased to more than twenty-eight times the voltage impressed on the terminals of the field before the switch was opened. Although the voltage applied to the motor field was only 31.5 volts the opening of the switch in the field circuit produced a transient stress of over 900 volts on the field insulation.

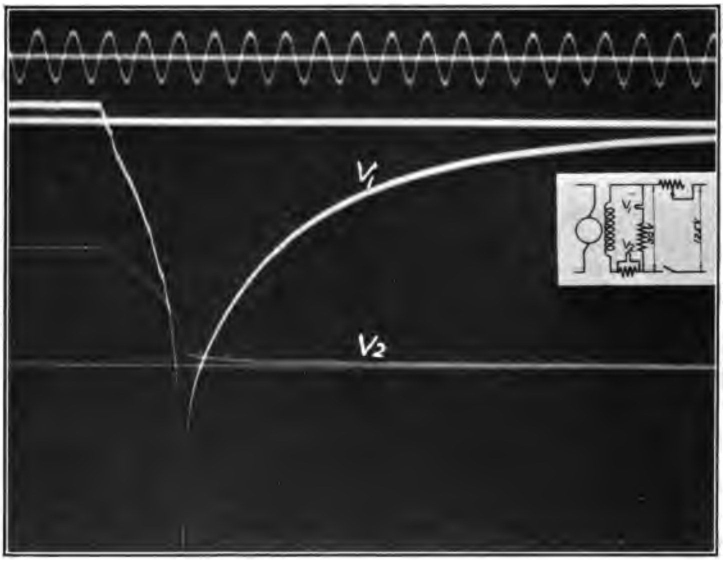

FIG. 30.—Breaking field circuit of direct current motor. Current and voltage transients.

Since the transient induced voltage on the motor field winding is directly proportional to the rate of cutting lines of force the shorter the time used in opening the switch, or the faster the resistance is inserted in the circuit the greater the transient voltage-stress tending to puncture the field insulation. If the circuit breaker operates in steps by which resistances of known value are introduced into the circuit in rapid succession the transient induced voltage will be proportionately lower and the destructive action of the arc greatly reduced. Since the energy stored in an

electromagnetic field depends on the current flowing in the field windings, it must be converted into some other form when the current is interrupted.

Current, Voltage and Magnetic Flux Transients.—In electromagnetic circuits having constant permeability the current, voltage and flux transients have the same shape and are expressed by the exponential equation. Referring to Fig. 27

$$e = Ri \tag{60}$$

The curve in the oscillogram therefore represents either the current or voltage transients and the quantitative values are obtained by applying the corresponding ampere and volt scales. From the law of electromagnetic induction the induced voltage is equal to the rate of cutting lines of force.

$$e = \frac{d\phi}{dt}10^{-8} \text{ volts} \tag{61}$$

Hence,

$$\phi = 10^8\int_0^t edt = \frac{EL10^8}{R}\epsilon^{-\frac{R}{L}t}$$

$$= \text{constant } \epsilon^{-\frac{R}{L}t} \text{ lines of flux} \tag{62}$$

The flux transient therefore is an exponential curve of the same form as the current and voltage transients.

In Fig. 31 is shown the corresponding transients for the current, voltage and flux in forming an electromagnetic field in a magnetic circuit of constant permeability. The transients are shown by the dotted lines, the permanent values by the broken lines and the instantaneous values by the full line curves.

$$i = I + \left(-I\epsilon^{-\frac{t}{T}}\right) = I - I\epsilon^{-\frac{R}{L}t} \tag{63}$$

$$e = E + \left(-E\epsilon^{-\frac{t}{T}}\right) = E - E\epsilon^{-\frac{R}{L}t} \tag{64}$$

$$\phi = \Phi + \left(-\Phi\epsilon^{-\frac{t}{T}}\right) = \Phi - \Phi\epsilon^{-\frac{R}{L}t} \tag{65}$$

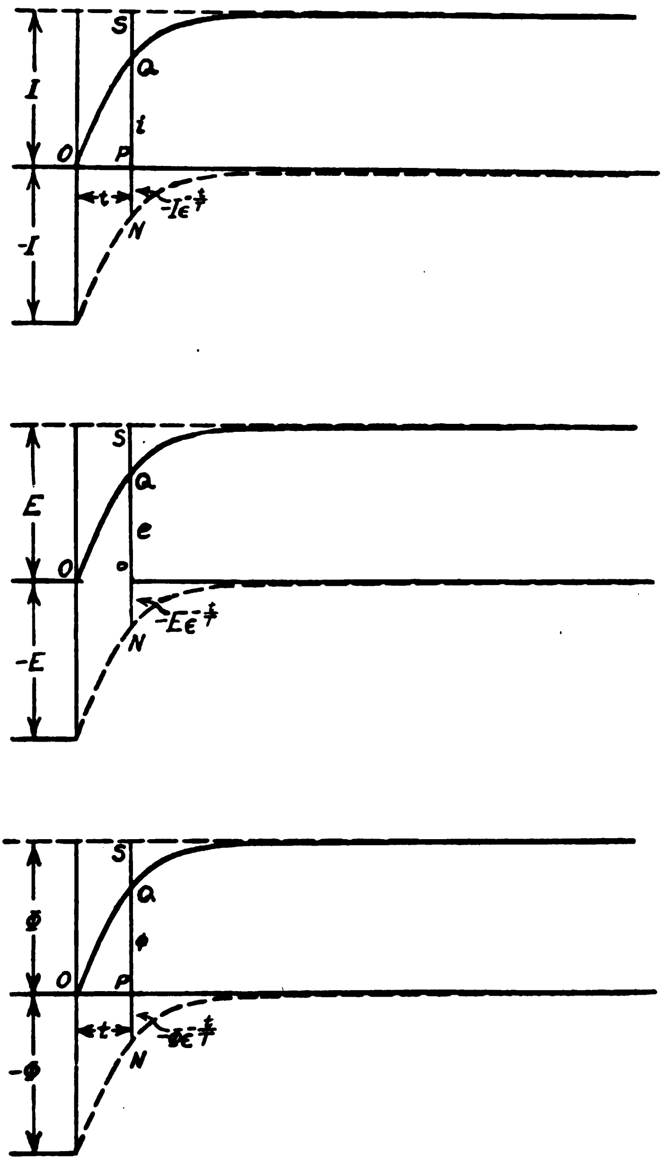

FIG. 31.—Single energy voltage, current and magnetic flux transients in forming a magnetic field in a magnetic circuit of constant permeability.

Equations (61), (62) and (63) express the instantaneous values as equal to the algebraic sum of the permanent and transient quantities. At any instant in time, t, as indicated in Fig. 31:

$$PQ = \text{the instantaneous value, } i, e \text{ or } \phi \tag{66}$$

$$PS = \text{the permanent or final value, } I, E \text{ or } \phi \tag{67}$$

$$PN = \text{the transient value, } I_0\epsilon^{-\frac{t}{T}}, E_0\epsilon^{-\frac{t}{T}}, \Phi_0\epsilon^{-\frac{t}{T}} \tag{68}$$

In discharging energy stored in a magnetic field, in a magnetic circuit of constant permeability, through a constant resistance, the transient and the instantaneous values are equal as the permanent value is zero.

$$i = I_0\epsilon^{-\frac{t}{T}} = I_0\epsilon^{-\frac{R}{L}t} \tag{69}$$

$$e = E_0\epsilon^{-\frac{t}{T}} = E_0\epsilon^{-\frac{R}{L}t} \tag{70}$$

$$\phi = \Phi_0\epsilon^{-\frac{t}{T}} = \Phi_0\epsilon^{-\frac{R}{L}t} \tag{71}$$

Problems and Experiments

1. A condenser of 115 mfds, charged to 500 volts, is discharged through a constant resistance of 425 ohms.

(*a*) Derive the equations for the current-time curve.

(*b*) Find the time constant of the circuit.

(*c*) Plot the voltage across the terminals of the condenser-time curve. Ordinates in volts and abscissae in seconds.

(*d*) Draw an ampere scale of ordinates so that the curve plotted in (*c*) will represent the current transient.

2. The time constant of an inductance coil is found by taking an oscillogram to be 0.04 seconds. The resistance in circuit was 15.8 ohms.

(*a*) Find the inductance in henrys.

(*b*) With 110 volts impressed on the coil plot the starting current transient.

(*c*) Write the equation for the current-time curve in (*b*).

3. Take an oscillogram of the starting current transient of the field of a laboratory motor or generator.

(*a*) From the oscillogram find the time constant of the field.

(*b*) Measure the resistance of the circuit and calculate the inductance of the field.

4. With the vibrators connected as shown in the circuit diagram in Fig. 30 taken an oscillogram showing the current and voltage transients produced by breaking the field circuit of a motor or generator.

5. By means of oscillograms determine the time required for the operation of automatic circuit breakers. Arrange the connection for the vibrators so as to show the time consumed by each step in the operation.

CHAPTER IV

SINGLE ENERGY TRANSIENTS. ALTERNATING CURRENTS

In direct-current systems the transient electric phenomena described in the preceding chapter, are due to the storage of energy in magnetic and dielectric fields. If a constant direct-current voltage is impressed on a circuit having constant resistance but neither inductance or condensance the current would instantly reach its permanent value, and any change in the voltage would at the same time cause a proportional change in the current. With either condensance or inductance in the circuit a short period of time is required for the current to reach its permanent value after any change in voltage, and the current-time curve during the transition period is expressed by the exponential equation. The value of the variable current is at any instant equal to the algebraic sum of the permanent and transient values; and single energy transients for direct currents in circuits having constant resistance and inductance or constant resistance and condensance, may be expressed by exponential equations similar in form to (63), (64) and (65).

Single Phase, Single Energy Load Circuit Transients.—The same principle applies to single energy transients in alternating-current systems. In circuits having constant resistance but neither inductance nor condensance, no transients appear. Any change in the voltage produces instantly a proportionate change in the current. In circuits having either inductance or condensance a transient period for the readjustment of the energy content of the system follows any change in voltage. At any instant the transient current or voltage is the algebraic sum of the

FIG. 32.—Single phase, single energy, current transient. $E = 129$ volts; $I = 0.59$ amps.; $R = 20.0$ ohms; $L = 0.575$ henrys; $f = 60$ cycles; $\gamma_1 = 90°$.

corresponding permanent and transient values. The permanent term, i' is the alternating current wave assumed to be of simple sine form as expressed in equation (72) with γ_1 as the time phase angle at the starting moment.

$$i' = \pm {}^{M}I \sin (\omega t - \gamma_1) \tag{72}$$
$$e' = \pm {}^{M}E \sin (\omega t - \gamma_2) \tag{73}$$

The transient term, i'' is expressed by the exponential equation of the same form as in direct currents with an initial value equal in magnitude but opposite in time phase to what would have been the permanent value at the starting point if the circuit has been closed at some previous time.

$$i'' = \pm {}^{M}I \sin \gamma, \epsilon^{-\frac{t}{T}} \tag{74}$$

$$e'' = \pm {}^{M}E \sin \gamma_2 \epsilon^{-\frac{t}{T}} \tag{75}$$

The instantaneous value of the current or voltage would therefore be expressed by (76) and (77):

$$i = i' + i'' = \pm {}^{M}I \sin (\omega t - \gamma_1) \pm {}^{M}I \sin \gamma_1 \epsilon^{-\frac{t}{T}} \tag{76}$$

$$e = e' + e'' = \pm {}^{M}I \sin (\omega t - \gamma_2) \pm {}^{M}E \sin \gamma_1 \epsilon^{-\frac{t}{T}} \tag{77}$$

The oscillogram in Fig. 32 shows the current-time curve, i, produced in a circuit having 20 ohms resistance and 0.575 henrys inductance when a 60 cycle alternating-current voltage, e, was impressed by closing a switch at the instant in time presented by the OY axis.

The voltage impressed is represented by the sine wave,

$$e = {}^{M}E \sin (\omega t - \gamma_2) \tag{78}$$

The actual current flowing is represented by the curve i, which practically coincides, after completing 6 cycles, with the permanent value i, shown by the dotted sine wave. The transient, i'', is shown as the broken line whose initial value,

$$ON = -OP = {}^{M}I \sin \gamma_1 \tag{79}$$

At any instant, t, after the closing of the switch, i is equal to the algebraic sum of i' and i''.

$$i = i' + i'' = {}^{M}I \sin(\omega t - \gamma_1) + {}^{M}I \sin \gamma_1 \, \epsilon^{-\frac{R}{L}t} \quad (80)$$

$$= 0.835 \sin\left(\omega t - \frac{\pi}{2}\right) + 0.835 \sin \frac{\pi}{2} \epsilon^{-35t} \quad (81)$$

It is evident that the initial value of the transient may vary from $-{}^{M}I$ to $+{}^{M}I$, depending at what point of the voltage-time curve the circuit is closed. If the switch be thrown at the instant when the permanent current wave would be zero, $\gamma = 0$, no transient would appear and the

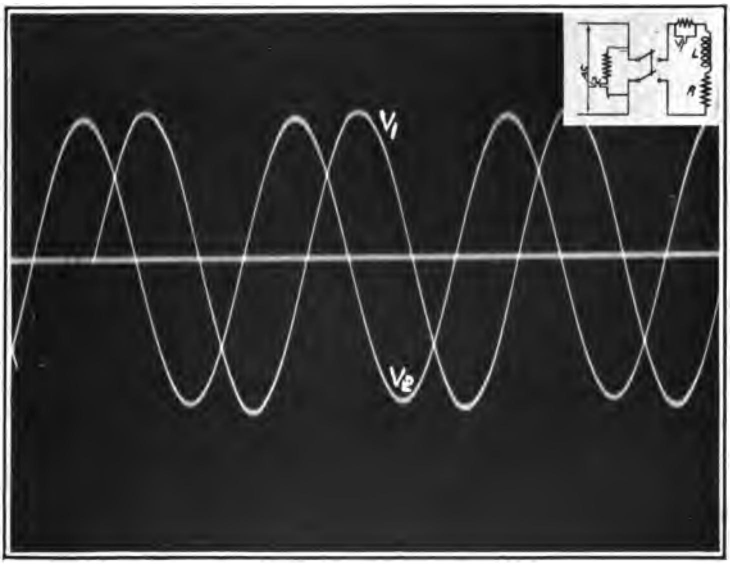

FIG. 33.—Single phase, single energy, current transient.
$E = 129$ volts; ${}^{M}E = 182.5$ volts; $I = 0.59$ amps.; ${}^{M}I = .835$ amps.; $R = 20$ ohms; $L = 0.575$ henrys; $f = 60$ cycles; $\gamma_1 = .0°$.

permanent and actual current time curves would coincide throughout as shown in Fig. 33.

$$i = i' = {}^{M}I \sin(\omega t) \quad (82)$$

The transient current would have a maximum initial value if the circuit is closed at the instant the permanent current wave is at a maximum, that is when sine $\gamma_1 = 90°$. The time constant for the current transient would be the

same at whatever point in the cycle the circuit is closed, as it depends on the resistance and inductance in the load circuit.

Three-phase, Single Energy, Load Circuit Transients.— For three-phase circuits similar relations exist. The starting current transients in three-phase systems in which energy may be stored either magnetically or dielectrically follow the same laws as discussed for single-phase circuits.

In Figs. 34 and 35 are shown oscillograms of the three starting load currents in a three-phase system, star connected and having 9.0 ohms resistance and 0.205 henrys inductance in each phase to neutral. The corresponding permanent current waves and transient currents were traced on the oscillogram in Fig. 34. In Fig. 35 the circuits were closed at the instant the current in v_1 was of zero value.

Phase 1:

Impressed voltage,

$$e_1 = {}^{M}E_1 \sin (\omega t - \gamma_2) \tag{83}$$

Permanent current,

$$i'_1 = {}^{M}I_1 \sin (\omega t - \gamma_1) \tag{84}$$

Initial value starting current transient,

$$OQ_1 = -OP_1 = {}^{M}I \sin \gamma_1 \tag{85}$$

Transient current,

$$i''_1 = {}^{M}I_1 \sin \gamma_1 \, \epsilon^{-\frac{R_1}{L_1}t} \tag{86}$$

Actual current, oscillogram,

$$i_1 = i'_1 + i''_1 = {}^{M}I_1 \sin (\omega t - \gamma_1) + {}^{M}I_1 \sin \gamma_1 \epsilon^{-\frac{R_1}{L_1}t} \tag{87}$$

Time constant,

$$T = \frac{L_1}{R_1} = 0.023 \text{ seconds} \tag{88}$$

Phase 2:

Impressed voltage,

$$e_2 = {}^{M}E_2 \sin (\omega t - \gamma_2 - 120°) \tag{89}$$

Permanent current,

$$i'_2 = {}^{M}I_2 \sin (\omega t - \gamma_1 - 120°) \tag{90}$$

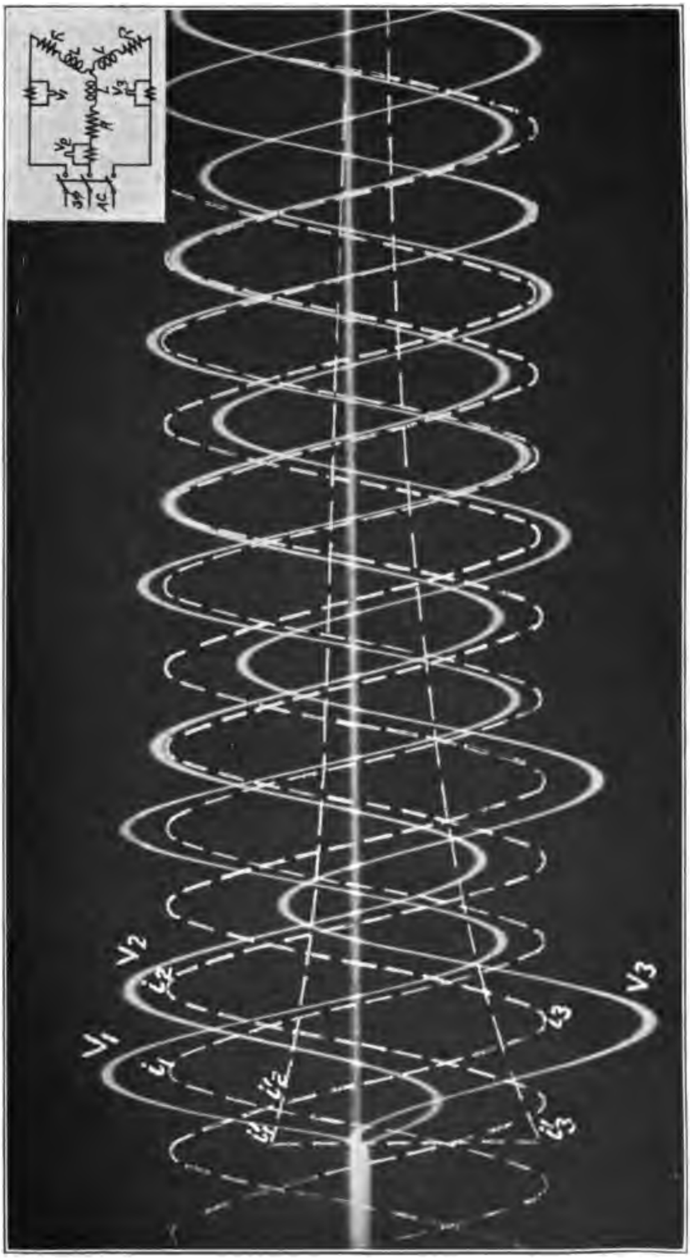

FIG. 34.—Three phase, single energy starting current transients. $E = 120$ volts; $R_1 = R_2 = R_3 = 9.0$ ohms; $L_1 = L_2 = L_3 = 0.205$ henrys.

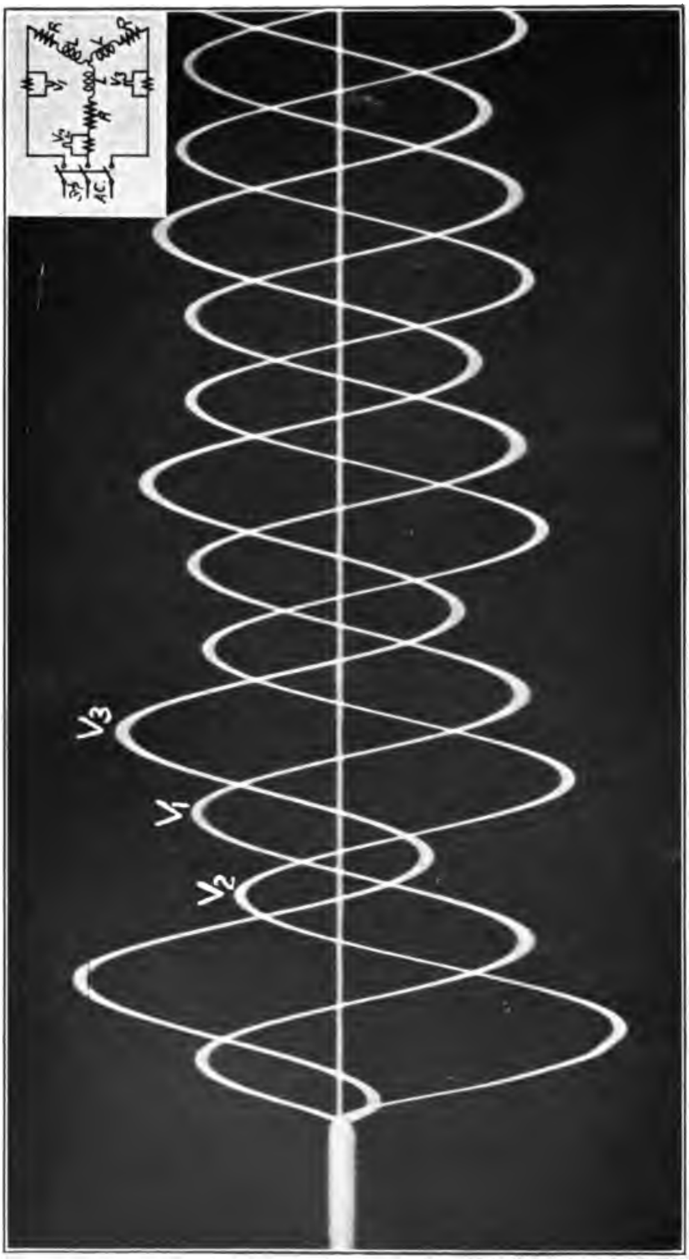

FIG. 35.—Three phase, single energy starting current transient. $E = 120$ volts; $R_1 = R_2 = R_3 = 9.0$ ohms; $L_1 = L_2 = L_3 = 0.205$ henrys.

Initial value starting current transient,

$$OQ_2 = -OP_2 = {}^{M}I_2 \sin (\gamma_1 - 120°) \tag{91}$$

Transient current,

$$i''_2 = {}^{M}I_2 \sin (\gamma_1 - 120°)\epsilon^{-\frac{R_2}{L_2}t} \tag{92}$$

Actual current, oscillogram,

$$i_2 = i'_2 + i''_2 = {}^{M}I_2 \sin (\omega t - \gamma_2 - 120°) + {}^{M}I_2 \sin (\gamma_1 - 120°)\epsilon^{-\frac{R_2}{L_2}t} \tag{93}$$

Time constant,

$$T_2 = \frac{R_2}{L_2} = 0.023 \text{ seconds} \tag{94}$$

Phase 3:

Impressed voltage,

$$e_3 = {}^{M}E_3 \sin (\omega t - \gamma_2 - 240°) \tag{95}$$

Permanent current,

$$i'_3 = {}^{M}I_3 \sin (\omega t - \gamma_1 - 240°) \tag{96}$$

Initial value starting current transient,

$$OQ_3 = -OP_3 = {}^{M}I_3 \sin (\gamma_1 - 240°) \tag{97}$$

Transient current,

$$i''_3 = {}^{M}I_3 \sin (\gamma_1 - 240°)\epsilon^{-\frac{R_3}{L_3}t} \tag{98}$$

Actual current, oscillogram,

$$i_3 = i'_3 + i''_3 = {}^{M}I_3 \sin (\omega t - \gamma_1 - 240°) + {}^{M}I_3 \sin (\gamma_1 - 240°)\epsilon^{-\frac{R_3}{L_3}t} \tag{99}$$

Time constant,

$$T_3 = \frac{L_3}{R_3} = 0.023 \text{ seconds} \tag{100}$$

It is of interest to note that the sum of the instantaneous values of the three currents, $i_1 + i_2 + i_3$, is equal to zero during the starting period as well as after the permanent state has been reached. This is evidently the case since under permanent conditions the sum of the currents is at any instant equal to zero and hence at the instant the circuit is closed, $OP_1 + OP_2 + OP_3 = 0$. Therefore, the sum of the initial values of the transient currents, $OQ_1 + OQ_2 + OQ_3 = 0$, and as the time constants of the three

transients are equal the sum of the actual currents in the three phases must at any instant be equal to zero.

Starting Transient of a Polyphase Rotating Magnetic Field.—In the preceding illustrations the single energy transients are due to changes in the amounts of energy stored in the given circuits, and the current-time curves show a continuous decrease of the current as expressed by the exponential equation. If the permanent condition relates to interconnected circuits which permit a transfer of energy from one circuit to another, although the total amount of energy stored in the magnetic field is constant, as in the rotating field of a polyphase induction motor, pulsations will appear during the transition period, that merit attention.

In Fig. 36 let a vector of constant length, ON, rotating in a counter clockwise direction represent a constant rotating magnetic field, as would be produced by three equal magnetizing coils, placed 120 deg. apart, and excited by three-phase currents, as, for example, in a three-phase induction motor. For simplicity let the rotor be removed and consider the stator circuit and the magnetic flux during the starting transition period in which the rotating field is built up to its constant permanent value. Let the switch, connecting the stator circuit to three-phase mains, be closed at the instant the rotating magnetic flux vector, ON, lies along the X axis, ON_0 in Fig. 36; as would have been the case if the switch had been closed at some previous time. The actual value of the rotating flux at the instant the circuit is closed is zero. The permanent value is represented by ON_0 and since the initial value of the transient flux must be equal in magnitude but of opposite time phase, it is represented by the vector OQ_0.

$$OQ_0 = (-ON_0) \tag{101}$$

From its initial values OQ_0 the transient flux decreases in magnitude, as indicated by the exponential flux-time curve in Fig. 37, but continues fixed in space direction

along the X axis. After the time, t_1, has elapsed, represented by the time angle N_0ON_1, the transient has a value OQ_1. The actual value of the flux must be the vector sum of the permanent value ON_1 and the transient OQ_1, or the resultant OP_1. In the time t_2, the transient has decreased to OQ_2 and the permanent flux vector reached the position ON_2. The actual flux OP_2 is the resultant of OQ_2 and ON_2. Similarly OP_3 is the resultant of OQ_3 and ON_3; OP_4, of

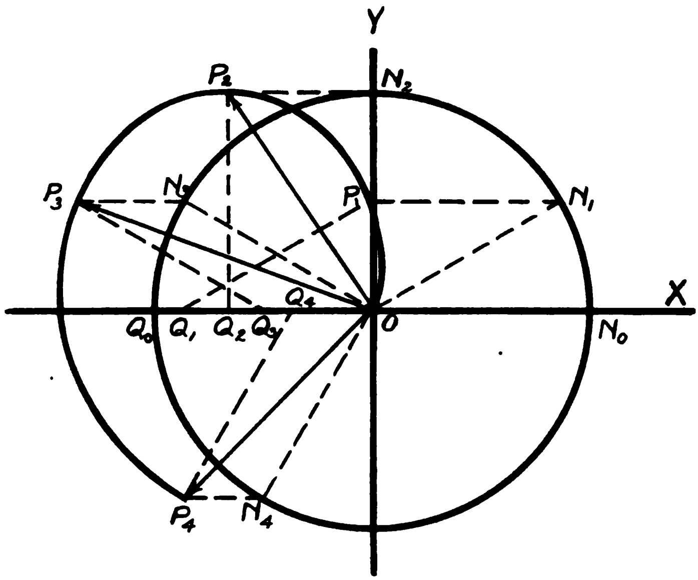

FIG. 36.—Permanent, transient and instantaneous values of the magnetic flux in starting a rotating magnetic field. Polar coordinates.

OQ_4 and ON_4, etc. From the vector diagrams, Figs. 36 and 38, it is evident that the actual starting flux will oscillate having values greater and smaller than the permanent value, the number of oscillations depending on the time constant of the circuit. The maximum value of the flux in the starting period will in any case be less than double the permanent value, as the transient flux continuously decreases from an initial value equal to the permanent flux in magnitude and different by 180 deg. in time-phase.

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The flux-time curve in Fig. 39 gives in rectangular coordinates the same relation as shown by the flux vector OP in the polar diagram in Fig. 38.

Polyphase Short Circuits. Alternator Armature and Field Transients.—Consider a three-phase alternator carrying a constant balanced load of constant power factor. The three-phase currents flowing in the armature produce a resultant constant armature flux or armature reaction.

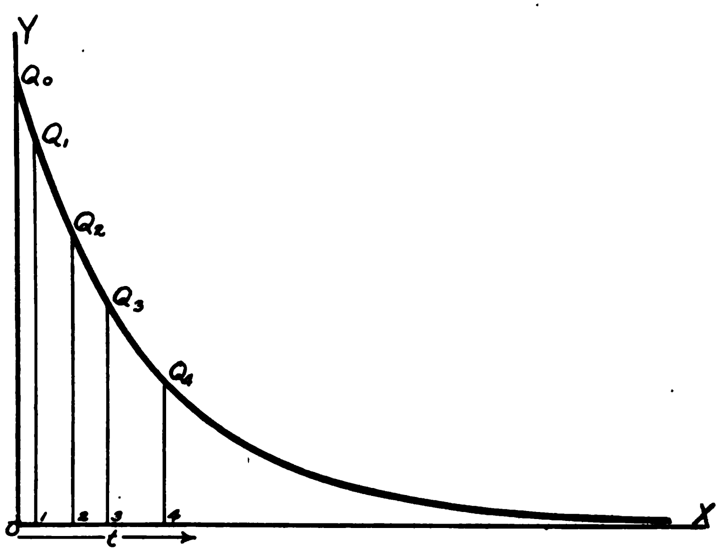

FIG. 37.—Starting magnetic flux transient from Fig. 36. Rectangular coordinates.

With respect to the field the armature flux is stationary but with respect to any diameter of the armature taken as a reference axis, the armature currents produce a constant rotating field of the same nature as the constant rotating field of a three-phase induction motor. For a machine in which the field is on the rotating spider while the armature is stationary, the resultant flux producing armature reaction rotates synchronously with the field. For an alternator with the field stationary and the armature rotating the resultant constant flux rotates at the same

speed but in direction opposite to the rotation of the armature and therefore is stationary with respect to the frame of the machine. In either case the armature flux or the armature reaction is stationary, if referred to the alternator field, but is a synchronously rotating field with respect to the armature.

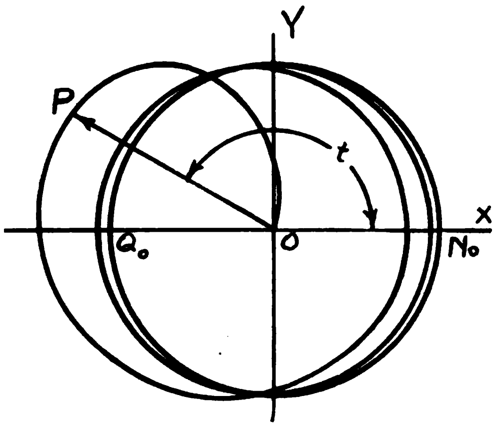

FIG. 38.—Starting a rotating magnetic field. Polar vector diagram of magnetic flux for three cycles of Fig. 36.

Due to the close proximity of the armature conductors to the field poles a large part of the magnetic flux produced by the armature currents passes through the field magnetic circuit. This causes a reduction in the field flux and therefore in the amount of energy stored magnetically by the exciting current in the field. Hence, although a constant direct-current voltage is impressed on the field circuit the useful flux is greatly reduced by the armature reaction, and as a consequence the generated armature voltage decreases in the same proportion. To effect any change in the amount of energy stored magnetically takes time and therefore the interaction of the armature flux with the field magnetic circuit produces electric transients.

During the transition period following the instant the short circuit occurs, two distinct causes are therefore superimposed in producing transient phenomena in the

interlinked electric and magnetic circuits of polyphase alternators.

(a) The armature transient which is equivalent to the starting transient of a rotating magnetic field including full frequency pulsations as illustrated in Figs. 38, 39.

(b) A field transient due to the reduction of the field flux by the armature reaction.

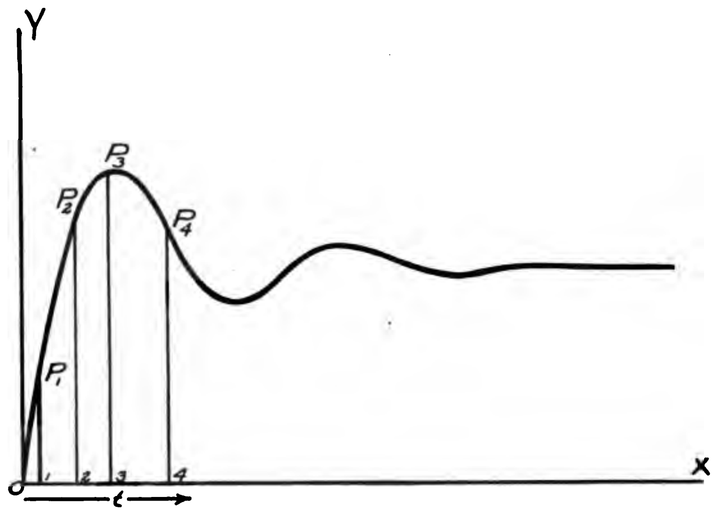

FIG. 39.—Same data as in Fig. 38. Rectangular coordinates.

The transients produced under (a) and (b) differ in duration, the ratio being in each case determined by the relative time constants of the armature and field circuits. In general the time constant in the field circuit is greater than in the armature circuits. Large turbo-alternators have very slow field transients as compared to the duration of the armature transients.

In Fig. 40 is shown an oscillogram of transients produced by a short circuit on all three phases of a 7.5 kw., 240 volt, 60 cycle, three-phase, star-connected alternator running idle and with 40 per cent normal field excitation. A similar oscillogram of short circuit transients for the same machine while carrying 50 per cent of full load is shown in Fig. 41.

As indicated in the circuit diagrams, Figs. 40 and 41, vibrator v_1 records the armature voltage across one pair of slip rings, e_a; vibrator v_2, the current, i_a, in one armature circuit, and vibrator v_3 the field current, i_f. As the short circuit is directly across v_1, the voltage e_a instantly drops to zero. The transient in the field winding is due to the combined action of the starting transient of the rotating field

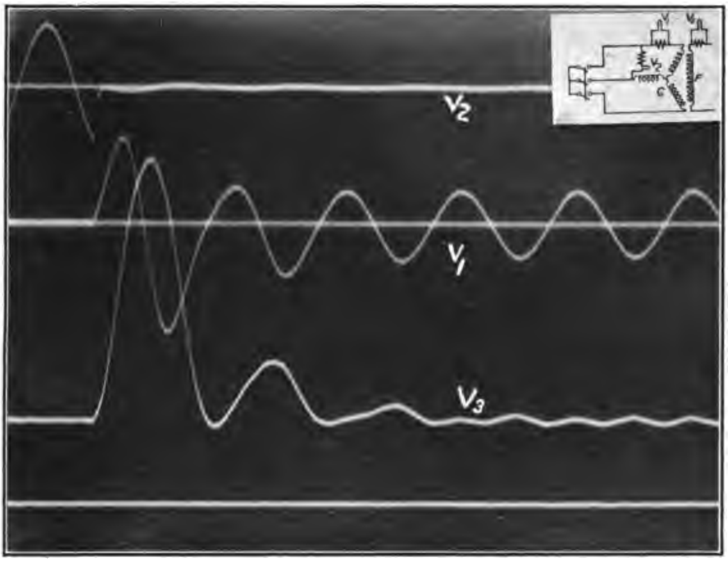

FIG. 40.—Short circuit transients from no load. Three-phase alternator, star-connected.
E, no load = 109 volts; I, short circuit = 12.0 amps.; I, field = 1.25 amps.; f = 60 cycles.

in the armature, which produces the full frequency pulsations, and the slower field transient resulting from the reduction of the field flux by the armature reaction. In breaking the short circuit the field transient alone will appear in the field winding, as shown by the oscillograms in Figs. 42 and 43. Necessarily the transient is reversed in direction from what is represented in Figs. 40 and 41, when the short circuit is made. It should be noted that breaking the armature short circuit was not instantaneous

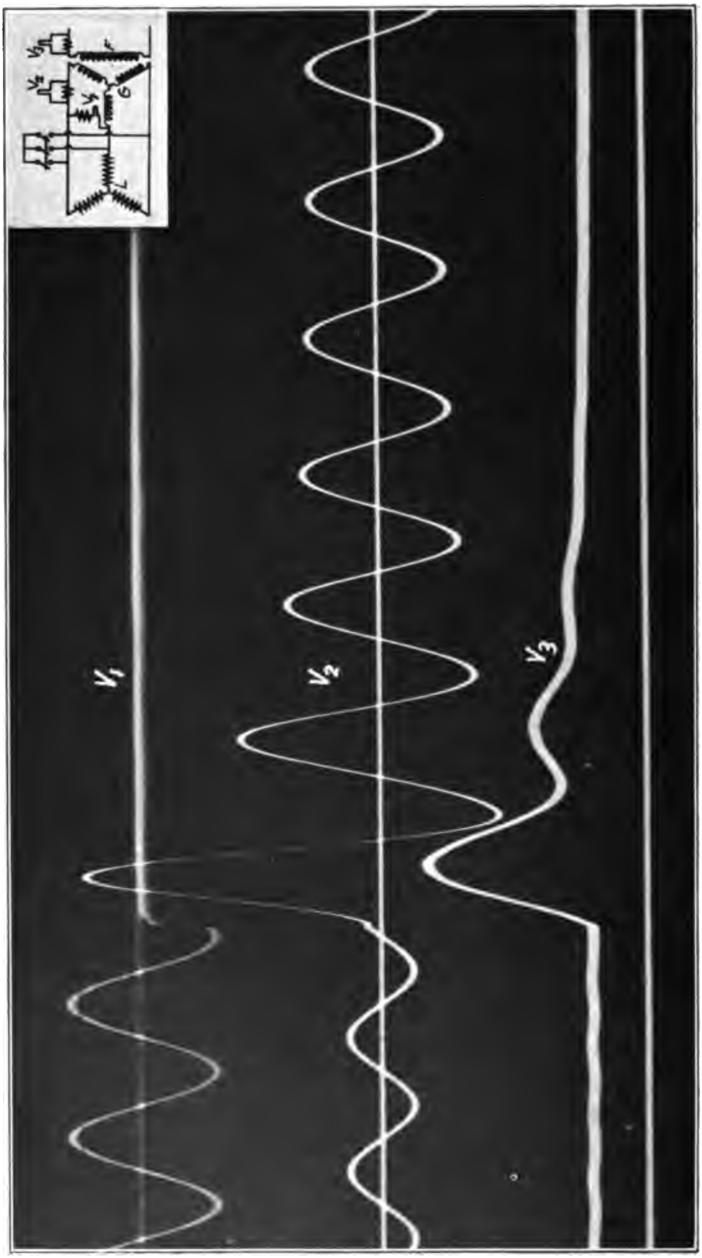

FIG. 41.—Short circuit transients from half load. Three-phase alternator, star-connected. E_1 load, = 136 volts; I_1 load, = 13.5 amps.; I_1 short circuit, = 19.8 amps.; I_1 field = 2.25 amps.; f = 60 cycles.

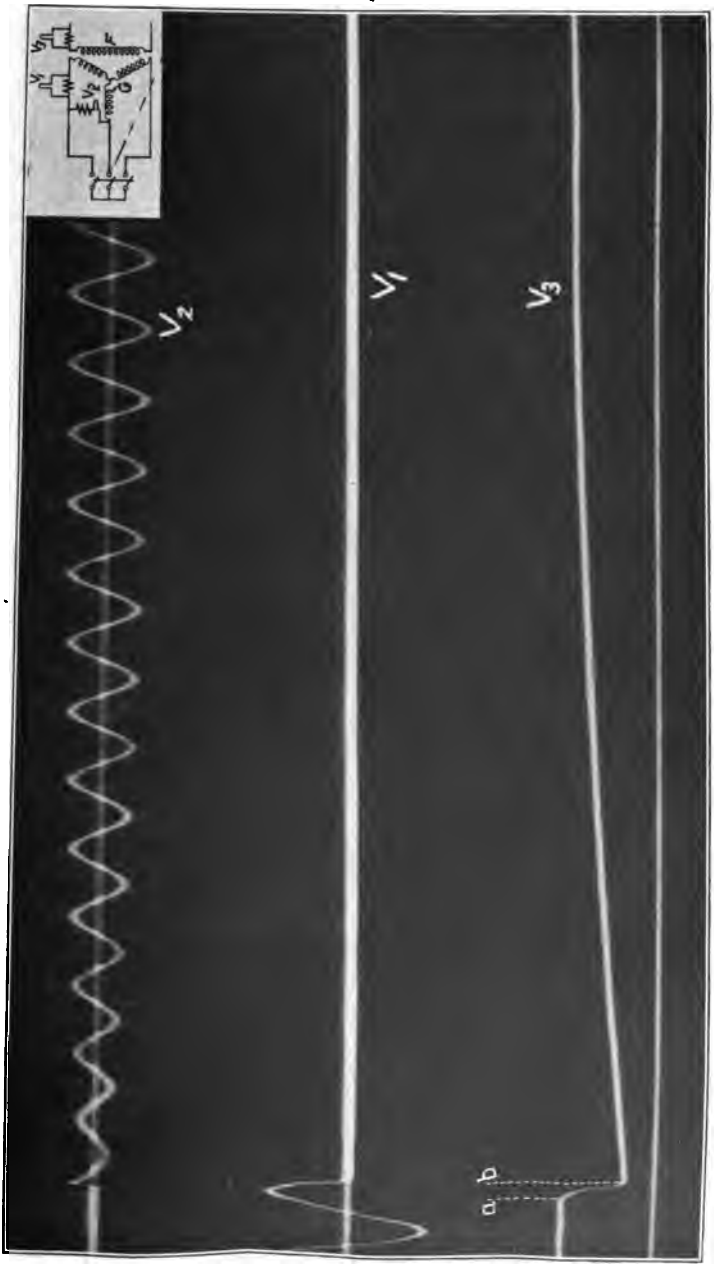

FIG. 42.—Transients in breaking short circuit. Three-phase, star-connected alternator, no load. E = 300 volts; I, short circuit = 31.0 amps.; I, field = 3.55 amps.; f = 60 cycles.

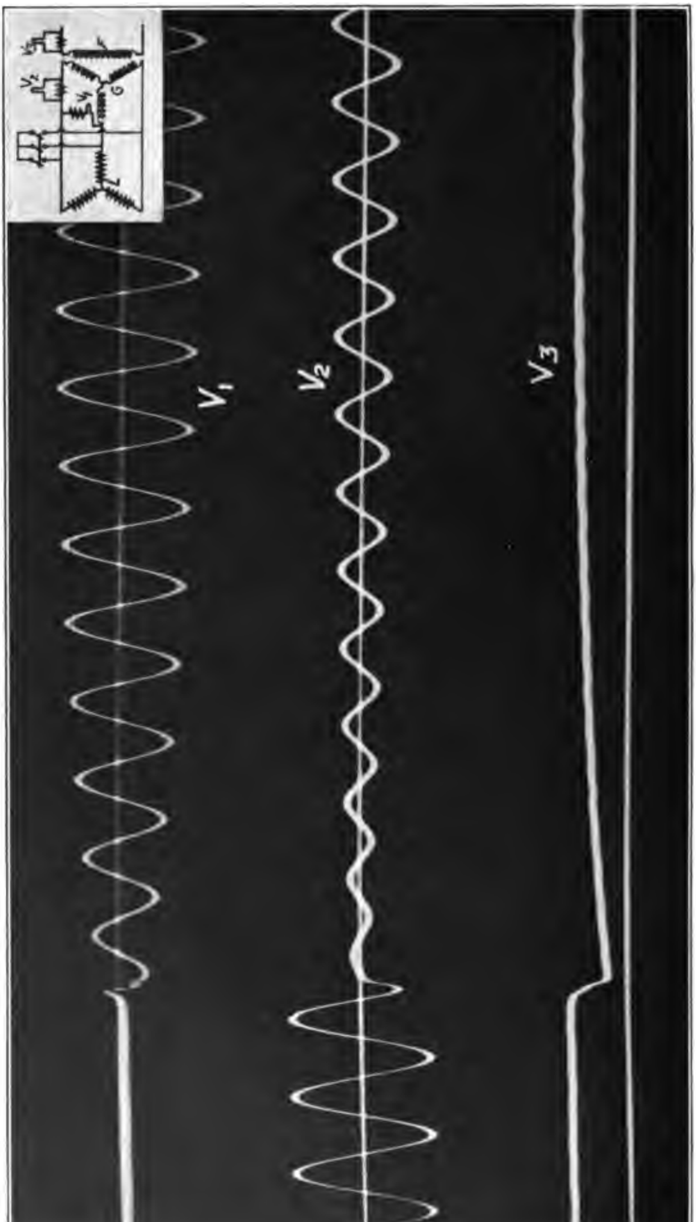

FIG. 43.—Transients in breaking short circuit. Three-phase, star-connected alternator, half load.

as arcs formed at the switch and continued the circuit during the time, $a - b$, Fig. 42, approximately for $\frac{1}{5}$ of a cycle or $\frac{1}{300}$ of a second. During this period the energy stored magnetically in the armature circuits was dissipated. Much more time, over 10 complete cycles, was required to restore full excitation in the field poles.

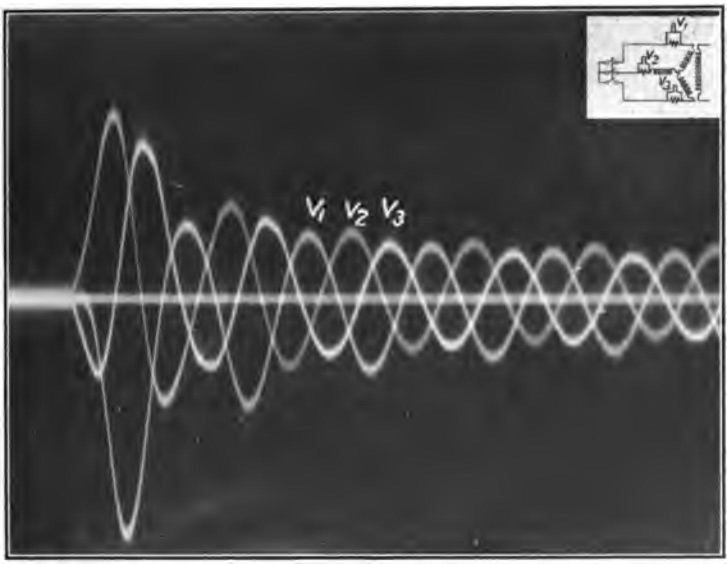

Fig. 44.—Armature current transients. Short-circuit on three-phase, star-connected alternator, no load.
E = 280 volts; I, short circuit = 28.7 amps.; I, field = 3.3 amps.; f = 59 cycles.

The direct relation of the voltage generated in the armature to the variable useful field flux is shown by the voltage wave, e_a, and the field transient, i_f, in Fig. 42. When the short circuit is made the same transients occur, but reversed in time, as is evident from oscillograms in Figs. 40, 41, 44 and 45.

In the operation of alternators the relative value of the initial or momentary to the final or permanent short circuit currents is of great importance. At any instant the short circuit current obeys Ohm's law, that is in magnitude it

will be directly as the voltage generated and inversely as the impedance of the armature circuit.

$$\dot{i}_a = \frac{e_a}{z_a} = \frac{e_a}{R_a + \dot{j}_{\iota}x_a} \tag{102}$$

Since the armature resistance, R_a, is small compared to the armature reactance ${}_{\iota}x_a$, equation (102) may be written as in (103).

$$\dot{i}_a = \frac{e_a}{\dot{j}_{\iota}x_a} \tag{103}$$

For constant speed the generated voltage, e_a, is directly proportional to the useful flux. At the instant the short circuit occurs and the alternator carries no load, as in Fig. 40, the useful flux depends on the direct current voltage impressed on the field winding and produces an armature voltage, ${}_oe_a$, and hence the initial or momentary value of the short circuit current,

$${}_o\dot{i}_a = \frac{{}_oe_a}{\dot{j}_ox_a} \tag{104}$$

If expressed in effective values as if the current sine wave continued at the initial magnitude,

$${}_oI_a = \frac{{}_oE_a}{{}_{\iota}x_a} \tag{105}$$

During the transient period following the short circuit the armature reaction reduces the field flux and as a consequence the voltage generated in the armature decreases in the same ratio. With the expiration of the field transient the useful flux, ϕ_u, is constant and hence the generated voltage, E_a, and the armature current, I_a, are constant or have permanent values.

$$I_a = \frac{E_a}{{}_{\iota}x_a} \tag{106}$$

$$\frac{I_a}{{}_oI_a} = \frac{E_a}{{}_oE_a} = \frac{\Phi_u}{{}_o\Phi_u} = \frac{\text{field excitation—armature reaction}}{\text{field excitation}} \tag{107}$$

Although the decrease in the armature current from its initial to the permanent value, as shown in Figs. 44 and 45,

is due to a reduction in the useful field flux and hence in the generated armature voltage it is customary to consider the voltage constant and ascribe the change to a fictitious increase in the reactance of the armature circuit. The combined effect of the armature reaction and the true armature reactance is represented by the so-called synchronous reactance ${}_s x_a$.

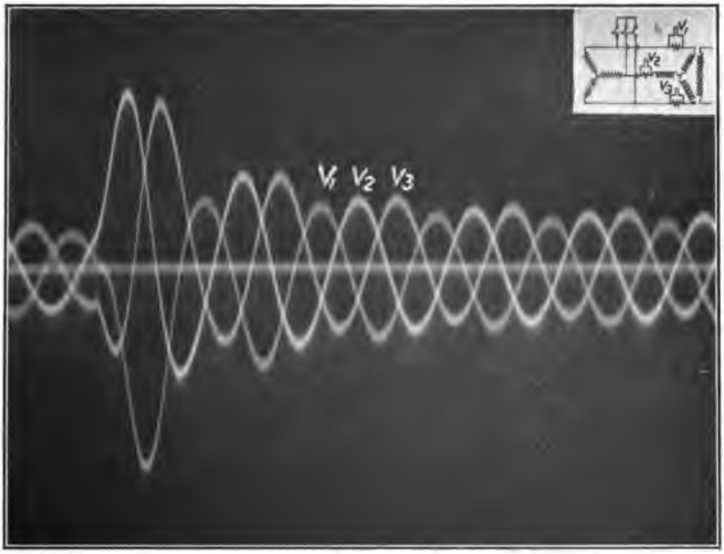

FIG. 45.—Armature current transients. Short-circuit on three-phase, star-connected alternator, 50 per cent. of full load.

The permanent short circuit current may therefore be expressed by equation (108) and the ratio of the permanent to the initial or momentary values by (109)

Let: I_a = permanent short circuit armature current.
${}_oI_a$ = initial short circuit armature current.
${}_L x_a$ = armature reactance.
${}_s x_a$ = synchronous reactance = armature reactance + armature reaction.

$$I_a = \frac{{}_oE_a}{{}_s x_a} \qquad (108)$$

$$\frac{I_a}{{}_oI_a} = \frac{{}_L x_a}{{}_s x_a} \qquad (109)$$

If it be assumed that the permeability of the magnetic circuits remains constant for the changes in flux density, the field current-time curve may also be expressed in the form of an equation in terms of the circuit constants and the initial value of the transients.

Let, $R_f =$ resistance of field circuit.
$L_f =$ inductance of field circuit.
$R_a =$ resistance of armature circuit.
$L_a =$ inductance of armature circuit.
$t =$ time from the instant short circuit occurs.
$\omega = 2\pi f$; $f =$ frequency in cycles per second.
$i_f =$ instantaneous value of field current.
$I_f =$ permanent value of field exciting current before short circuit occurs.
$i'_f =$ instantaneous value of current in field circuit due to field transients.
$I'_f =$ initial value of i'_f.
$i'_{af} =$ instantaneous value of current in field circuits due to armature transient.
$I'_{af} =$ initial value of i'_{af}

$$i'_f = I'_f \epsilon^{-\frac{R_f}{L_f}t} \tag{110}$$

$$i'_{af} = I'_{af} \epsilon^{-\frac{R_a}{L_a}t} \sin(\omega t) \tag{111}$$

In Figs. 42, 43:

$$i_f = I_f - i'_f = I_f - I'_f \epsilon^{-\frac{R_f}{L_f}t} \tag{112}$$

In Figs. 40, 41:

$$i_{af} = I_f + i'_f + i'_{af} = I + I'\epsilon^{-\frac{R^f}{L_f}t} + I'\epsilon^{-\frac{R_a}{L_a}t} \sin \omega t \tag{113}$$

Short circuit currents, particularly under normal field excitation, produce so great changes in flux density that the permeability is not constant and hence L_a and L_f are not constant. The purpose of the equation is however, merely to state in concise form the factors involved without taking into consideration the complications due to variations in the permeability of the magnetic circuits.

While short circuits produce electric transients of greater magnitude than the changes that occur during normal operation of alternators, it should be kept in mind that any modification in the armature currents, as, for example, an increase or decrease in the load, produces transients having the same characteristics as those produced by short circuits. Any change in the amount of energy stored magnetically in the armature or field circuits requires time and during the period of readjustment electric transients are produced in the interlinked electric and magnetic circuits.

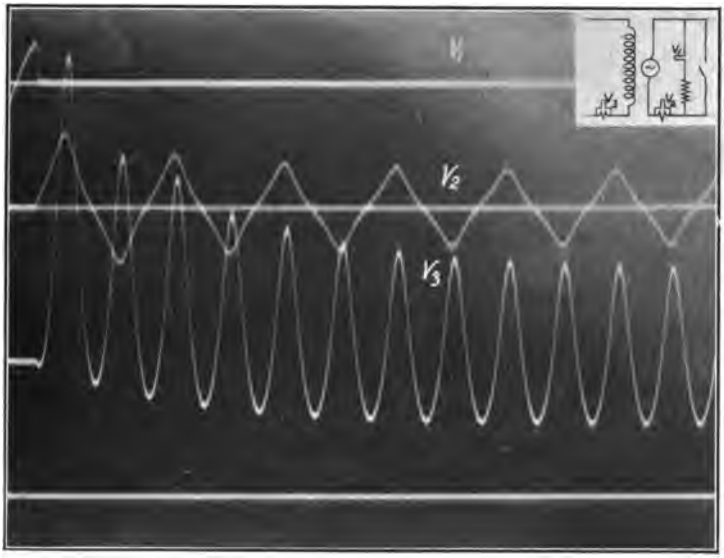

Fig. 46.—Short circuit transients, single phase alternator. Symmetrical. No load.
E = 108 volts; I, short circuit = 10.4 amps.; I, field = 2.6 amps.; f = 60 cycles.

Single-phase Short Circuits. Alternator Armature and Field Transients.—In polyphase alternators the permanent armature field produced by the balanced armature currents, and hence the armature reaction, is constant in value and, with respect to the alternator field poles, fixed in position. In single-phase alternators the magnetic field produced by the armature currents, and therefore the armature reaction,

pulsates synchronously with the armature rotation. The pulsations of the armature reaction necessarily appear in the field circuit. As the armature rotates 180 electrical degrees for each half cycle of the armature current, the pulsations of the armature reaction with respect to the field poles will have double the frequency of the armature currents. Therefore, the field current has a permanent double frequency pulsation as shown in Figs. 46 and 47.

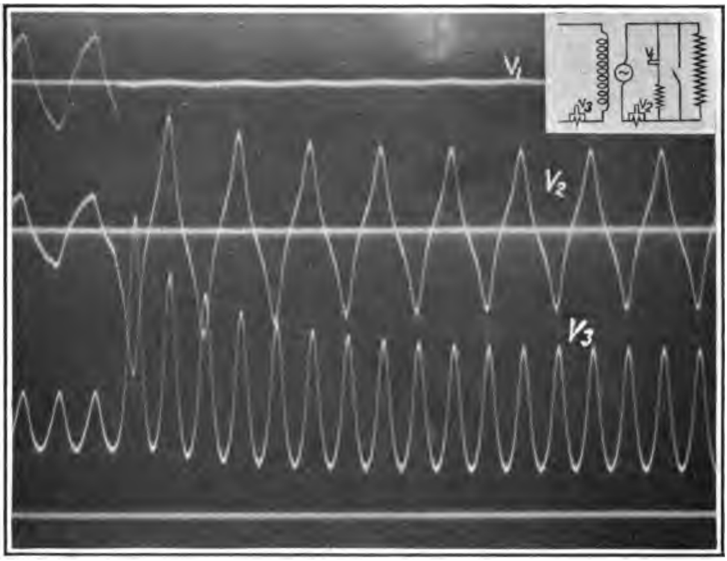

FIG. 47.—Short circuit transients, single phase alternator. Symmetrical. Load. E, load = 106 volts; I, load = 16.83 amps.; I, short circuit = 21.5 amps.; I, field = 2.6 amps.; f = 60 cycles.

Since the armature reaction is pulsating and not constant, as in polyphase alternators, the initial value of the starting transient of the armature flux will depend on the point on the current wave at which the short circuit occurs. Thus in Figs. 46 and 47 the short circuiting switch closed nearly at the instant the armature current was zero and hence only a very small armature transient was produced. With the armature transient absent the field current-time oscillograms, as illustrated in Figs. 46 and 47, are symmetrical

showing the permanent double frequency pulsations of the armature reaction superimposed on the field transient.

If the short circuit occurs at other than the zero points on the armature current wave, an armature transient of full frequency is produced for the same reason as explained for short circuits in polyphase alternators. The oscillograms in Figs. 48 and 49 show the asymmetrical field current-

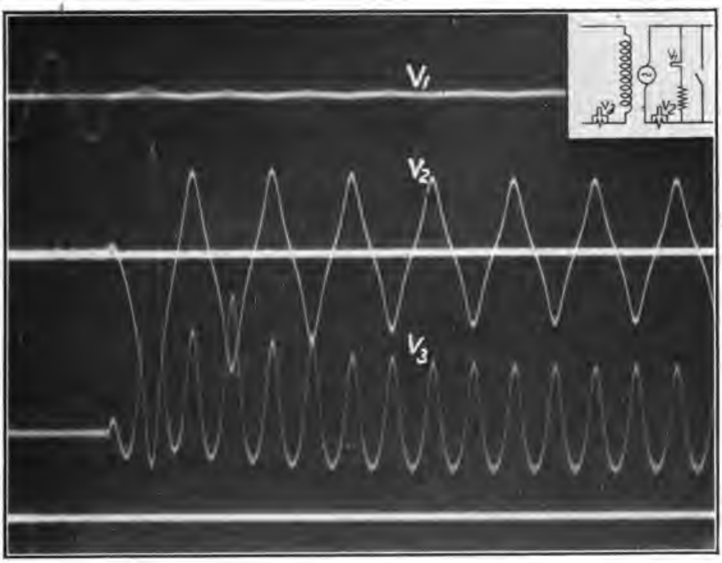

FIG. 48.—Short circuit transients, single phase alternator. Asymmetrical. No load.
E = 57 volts; I, short circuit = 23.0 amps.; I, field = 1.3 amps.; f = 60 cycles.

time curves on which are superimposed the double frequency permanent armature reaction, the field transient, and the full frequency pulsation produced by the armature transient. The combination of the full frequency armature transient pulsation with the permanent double frequency armature reaction produces the asymmetry in the curves. The ordinates for the odd numbers of the double frequency waves add to the full frequency values, while for the even number of waves the difference in the ordinates produces the wave recorded by the oscillograph.

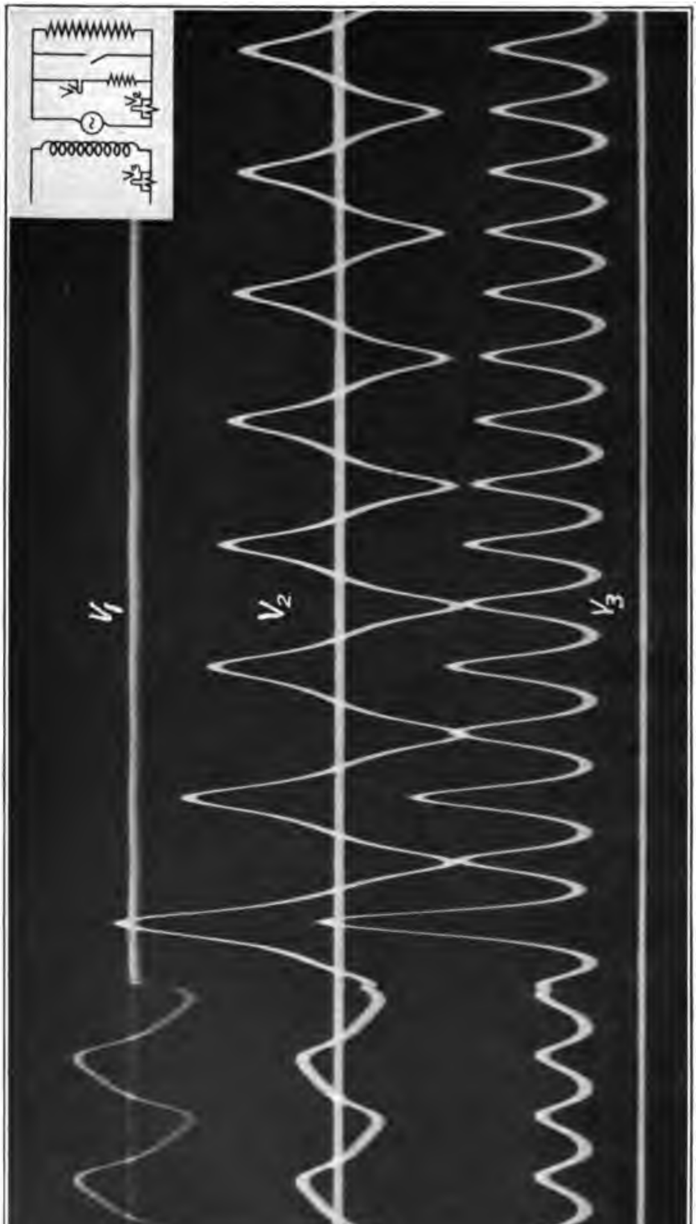

FIG. 49.—Short circuit transients, single phase alternator. Asymmetrical. Load.

Hence, during the transition period the peaks of the odd numbered waves decrease, while the even numbered peaks increase, and at the expiration of the armature transient all reach the permanent constant pulsation produced by the pulsating armature reaction. While the field current pulsates as a result of the double frequencyarmature reaction and the full frequency armature transients, the voltage across the field terminals will pulsate to a greater or less degree depending on the amount of external resistance and inductance in series with the field circuit. With much external resistance or impedance the voltage at the terminals of the field winding may reach high values which may puncture the insulation and cause a short circuit in the field exciting circuit.

The field transient separated from the armature reaction may be shown by taking an oscillogram of the field current when the short circuit on the single phase alternator is broken, as shown in Figs. 50 and 51. The armature transient is dissipated during the opening of the switch, indicated by the time $a - b$ on the oscillogram, while several complete cycles are required before the field flux, and as a consequence the armature voltage, regains its full value. In the transition period following the closing or opening of the short circuiting switch the oscillograms of the field currents show the effects of the energy changes taking place in both the field and armature circuits.

Under the assumption that the permeability of the magnetic circuits is constant the field-current-time curve in Figs. 46 to 51 may be expressed in terms of the circuit constants and the initial values of the transients:

Let: R_f = resistance of field circuit.
L_f = inductance of field circuit.
R_a = resistance of armature circuit.
L_a = inductance of armature circuit.
t = time from the instant short circuit occurs.
$\omega = 2\pi f$; f = frequency in cycles per second.
i_f = instantaneous value of field current.

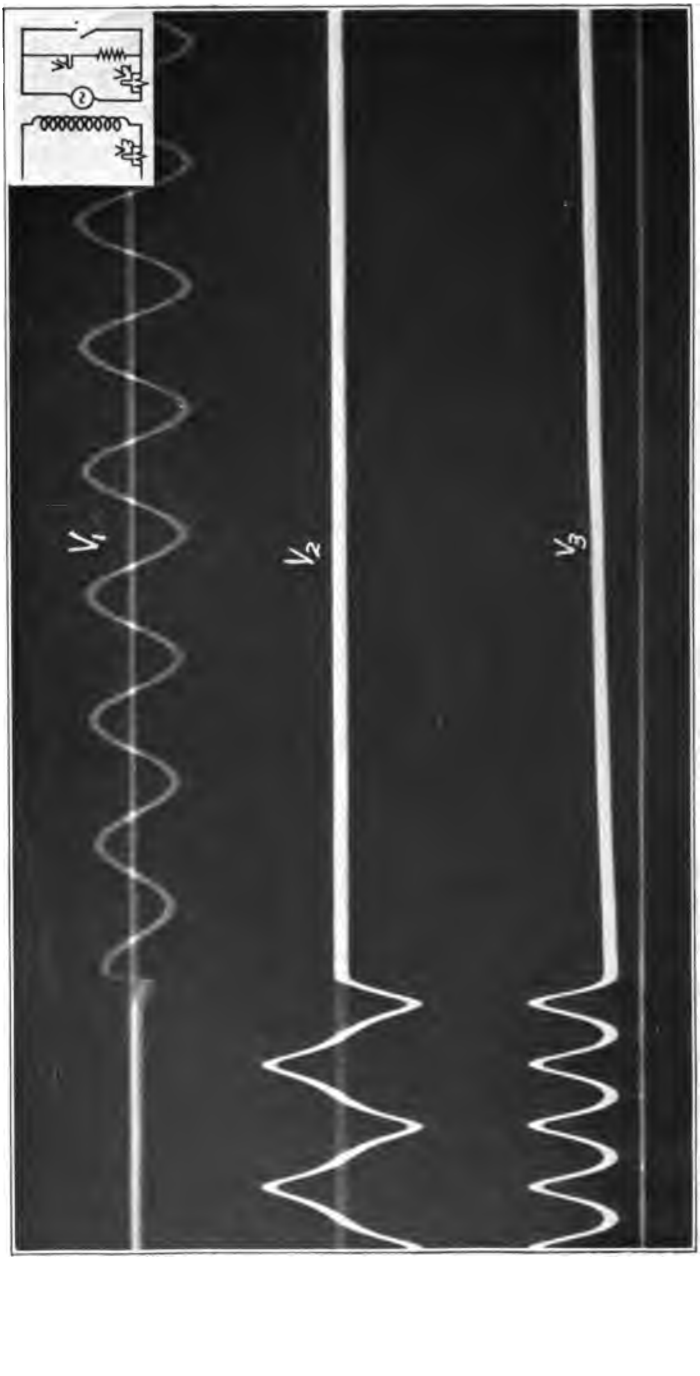

Fig. 50.—Breaking short circuit, single phase alternator. No load.

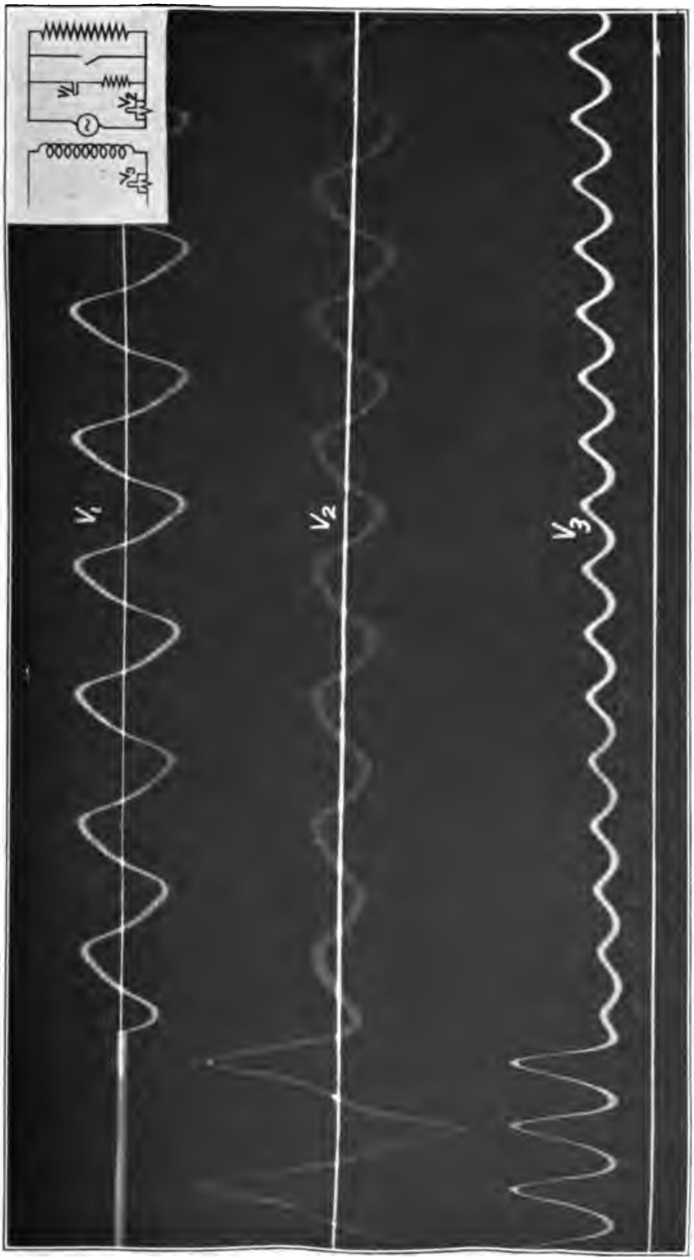

FIG. 51.—Breaking short circuit, single phase alternator. Load. E, no load = 315 volts; E, full load = 220 volts; I, load = 19.3 amps.; I field = 322 amps.; f = 60 cycles.

I_f = permanent value of exciting current before short circuit occurs.
i'_f = instantaneous value of current in field circuit due to field transient.
I'_f = initial value of i'_f.
i_{af} = instantaneous value of current in field circuit due to armature reaction.
I_{af} = maximum value of i_{af}.
i'_{af} = instantaneous value of current in field circuit due to armature transient.
I_{af} = maximum initial value of i'_{af}.
γ_1 = phase angle of i_{af}.
γ_2 = phase angle of i'_{af}.

$$i_f = I'_f \epsilon^{-\frac{R_f}{L_f}t} \tag{114}$$

$$i_{af} = {}^{M}I_{af}{}^{M} \sin (2\omega t - \gamma_1) \tag{115}$$

$$i'_{af} = {}^{M}I'_{af}\epsilon^{-\frac{R_a}{L_a}t} \sin (\omega t - \gamma_2) \tag{116}$$

In Fig. 50:

$$i_f = I^f - I_f'\epsilon^{-\frac{R_f}{L_f}t} \tag{117}$$

In Fig. 46:

$$i_f = I_f + I'_f\epsilon^{-\frac{R_f}{L_f}t} + {}^{M}I_{af} \sin (2\omega t - \gamma_1) \tag{118}$$

In Fig. 48:

$$i_f = I_f + I'_f\epsilon^{-\frac{R_f}{L_f}t} + {}^{M}I_{af} \sin (2\omega t - \gamma_1) + {}^{M}I\epsilon^{-\frac{R_a}{L_a}t} \sin(\omega t - \gamma_a) \tag{119}$$

As indicated by the difference in the upper and lower halves of the double frequency pulsation the permeability of the magnetic circuit changed with the flux density. Under full field excitation the short circuit transients would produce much greater changes in the flux density and hence in the permeability of the steel in the armature and field poles. For this reason the equations are not directly applicable to commercial problems but state the

relations of the factors involved provided the permeability of the iron core is constant.

Single-phase Short Circuit on Polyphase Alternators.— If all phases of polyphase alternators are short circuited simultaneously the armature transients appear in the field circuit as full frequency pulsations produced by the rotating magnetic field, as illustrated for three-phase machines in Figs. 40 to 43.

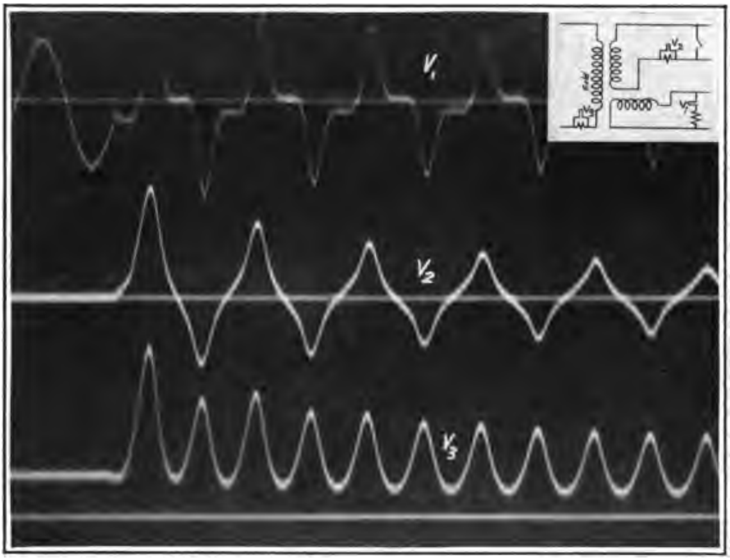

FIG. 52.—Single phase short circuit on three phase alternator.

If one phase only is short circuited the effect on the field circuit is essentially the same as illustrated for single phase alternators in Figs. 46 to 48. In Fig. 52 is shown the transient of the field current of a three-phase alternator after short circuiting one phase. The field current-time curve shows the effects produced by the field and armature transients and the permanent double frequency pulsations due to the armature reaction. In Fig. 53 is shown an oscillogram for a single-phase short circuit on a three-phase alternator which after 4 cycles is followed by a short circuit

on all three phases. While only one phase is short circuited the field current shows the double frequency pulsations combined with both the armature and field transients. After the three-phase short circuit occurs the field current shows the full frequency pulsations of the armature transient combined with the slower field transient.

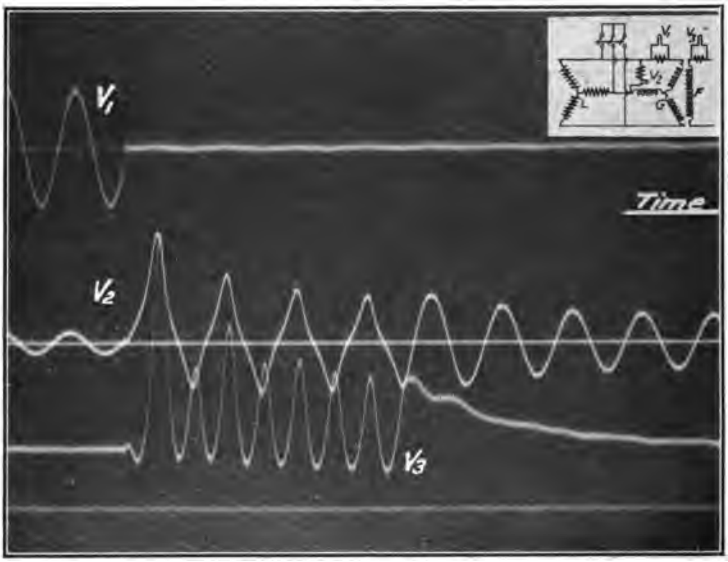

FIG. 53.—Single phase short circuit on three phase alternator followed by a three phase short circuit.

E = 118 volts; I, load = 10 amps.; I, short circuit = 22.5 amps.; I, field = 2.6 amps.

Oscillograms of transients in polyphase systems produced by single-phase short circuits necessarily differ with the type of machine and the way the transient magnetic fluxes interlink with the electric circuit to which the oscillograph vibrator is connected. Thus in Fig. 54 the open phase voltage of a two-phase alternator with short circuit on one phase shows a triple frequency harmonic, while the field current shows the double frequency pulsation combined with the field and armature transients of the same characteristics as for single-phase alternators.

FIG. 54.—Single phase short circuit on two phase alternator.

Problems and Experiments

1. Let the sine wave curve in Fig. 55 represent the 60 cycle alternating current that would flow in a circuit having 3.0 ohms resistance, 0.05 henrys inductance for a given voltage.

(*a*) Let the switch impressing the voltage on the circuit be closed at the instant marked (*a*) in the diagram. Draw in rectangular coordinates:

1. The permanent current sine wave as in Fig. 55.
2. The starting transient.
3. The actual current flowing in the circuit during the first $\frac{1}{5}$ second after the switch is closed.

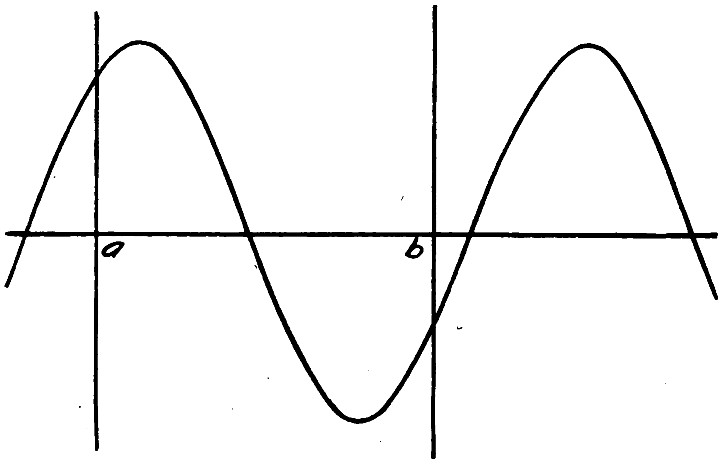

FIG. 55.—Single phase current, sine wave, 60 cycle starting transient.

(*b*) Similar to (*a*) except the voltage is impressed at the instant marked (*b*).

2. In a circuit having 60 ohms resistance and 0.045 henrys inductance a 25 cycle current is flowing, as represented by the sine wave on the left side in Fig. 56. At the instant marked (*a*) the impressed voltage is suddenly changed so that it will produce a permanent 60 cycle current shown by the dotted line sine wave in the figure.

(*a*) Draw in rectangular coordinates:

1. The sine current waves as in Fig. 56.
2. The starting transient.
3. The actual 60 cycle current for the first $\frac{1}{10}$ second after the voltage was changed.

(*b*) Same as (*a*), except the change is made at some other point along the time axis.

3. Take an oscillogram of the starting current in a circuit of known resistance and inductance. Calculate the starting transient and draw it on the oscillogram. Check by combining the ordinates for the actual current

recorded by the oscillograph with the corresponding values of the calculated transient and compare the resulting curve with the permanent current sine wave.

4. Let the sine waves in Fig. 57 represent the permanent value of the currents flowing in a balanced three-phase system, whose time constant is $\frac{1}{1,000}$ of a second.

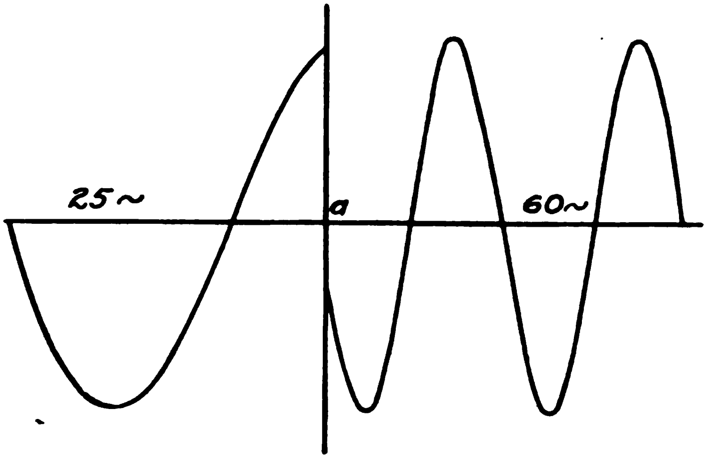

Fig. 56.—Single phase current, sine wave, 25 cycles to 60 cycles transient.

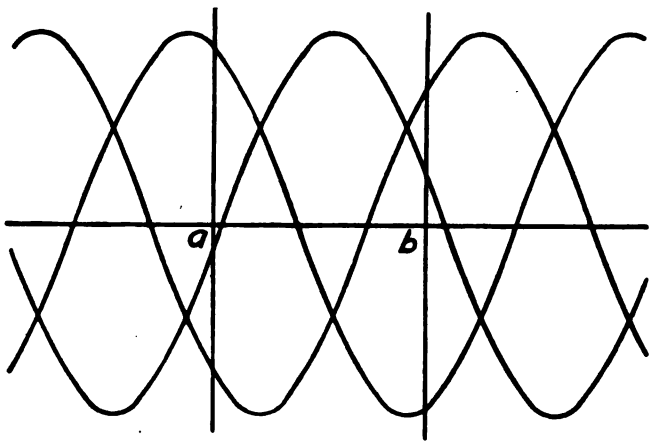

Fig. 57.—Three phase current, sine wave, 60 cycle starting transients.

(*a*) Let the voltage be impressed at the instant marked (*a*). Draw in rectangular coordinates:

1. The permanent current sine waves as in Fig. 57.
2. The starting transients for the three phases.

3. The actual currents as would be recorded by an oscillograph if a vibrator was connected to each of the three phases so as to record the current-time curves.

(b) Same as (a), except the voltage is impressed at the instant marked (b).

5. Take an oscillogram of the starting currents in a three-phase system connecting the vibrators as in the circuit diagram in Fig. 34. From the oscillogram and the circuit constants plot the starting transients and check with the permanent current waves as explained in Prob. 3.

6. Make oscillograms similar to Figs. 40 and 42 or 41 and 43. Obtain the necessary data to draw the scale in amperes or volts for each vibrator. A circuit diagram showing the position of each vibrator should be attached to each film.

7. Make oscillograms similar to Figs. 46 and 48 or 47 and 49. Quantitative data should be obtained for each vibrator and for the circuit constants.

8. Make an oscillogram similar to Fig. 53.

CHAPTER V

DOUBLE ENERGY TRANSIENTS

Single energy transients occur in electric circuits or other apparatus in which energy can be stored in only one form. Any change in the amount of energy stored produces transients and whether the stored energy is decreased or increased the transient itself is a decreasing function with its maximum value at the first instant. In magnetic, electric and dielectric circuits in which the resistance, inductance and condensance are constant during the transition period, single energy transients may be expressed by the exponential equation as discussed in Chaps. III and IV.

In apparatus having two forms of energy storage as a pendulum or electric circuits having both inductance and condensance, a series of oscillations may take place by which the energy is transferred from one form to the other, while the dissipation of the stored energy into heat proceeds in much the same manner as in single energy systems. Thus a pendulum, freely suspended in air, will swing back and forth over arcs of decreasing amplitude, with energy changing from the kinetic to the potential form and back to the kinetic twice for each cycle. The amplitude of each swing is less than for the one preceding since part of the energy has been dissipated into heat by friction during the intervening time. The pendulum comes to rest when all the stored energy is dissipated into heat.

In electric circuits having both dielectric and magnetic storage facilities the energy stored in one form may change to the other and back and forth in a series of oscillations of definite frequency. This is illustrated by the oscillogram in Fig. 58. The energy stored in a condenser is discharged through a resistance in series with an inductance. In

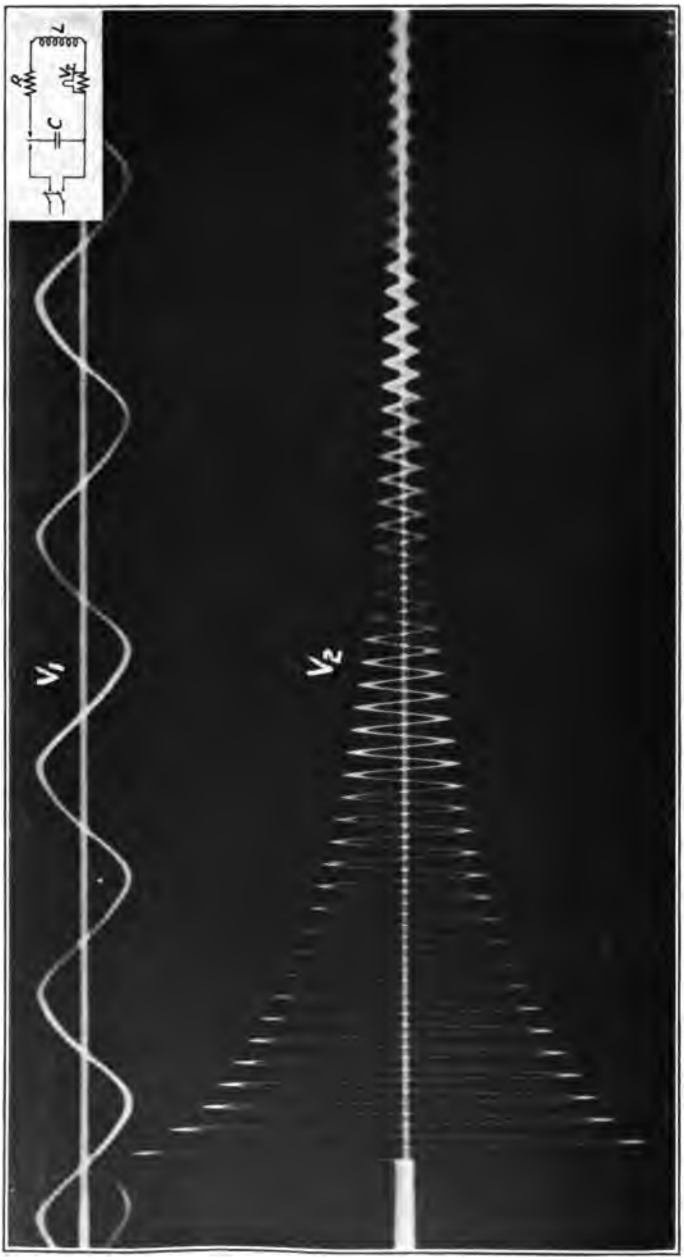

Fig. 58.—Double energy transient in a simple circuit. E = 500 volts; R = 7.8 ohms; G = 0; L = 89 milli-henrys; C = 0.25 microfarads; timing wave 100 cycles; f = 1070 cycles.

passing from the dielectric field to the magnetic field or the reverse, the energy goes through the resistance and a part is dissipated by the Ri^2 losses. Hence the amplitude of each oscillation is less than for the one preceding. By referring to the timing wave on the oscillogram, Fig. 58, it is found that the frequency of oscillation was 1,070 cycles per second, and that practically all the energy was dissipated into heat by the Ri^2 losses in 50 cycles, or approximately $1/20$ of a second.

Surge or Natural Impedance and Admittance.—If no energy is dissipated during the transfer the stored energy in the dielectric field when the voltage is a maximum must be equal to the quantity stored in the magnetic field when the current is a maximum. Hence from (7) and (16)

$$\frac{C{}^{m}E^2}{2} = \frac{L{}^{m}I^2}{2} \tag{120}$$

Therefore, from (120)

$$\frac{{}^{m}E}{{}^{m}I} = \sqrt{\frac{L}{C}} = {}_{n}z, \text{ the } \textit{surge} \text{ or } \textit{natural} \text{ impedance of the circuit} \tag{121}$$

$$\frac{{}^{m}I}{{}^{m}E} = \sqrt{\frac{C}{L}} = {}_{n}y, \text{ the } \textit{surge} \text{ or } \textit{natural} \text{ admittance of the circuit} \tag{122}$$

The quantity, $\sqrt{L}/\sqrt{C}$, is in the nature of an impedance and is called the surge or natural impedance of the circuit, and its reciprocal, $\sqrt{C}/\sqrt{L}$, the natural or surge admittance of the circuit.

Frequency of Oscillation in Simple Double Energy Circuits.—Consider circuits "*a*" and "*b*" in Fig. 59. Let the inductance, L, the condensance, C, and the resistance, R, be constant and of the same value in the two cases. Let an alternating current voltage be impressed on the terminals and let the frequency be varied until the current is in phase with the voltage at the terminals. All the energy absorbed by the Ri^2 losses is supplied from the a.c. mains.

In circuit (*a*) under the given conditions:

$$j_L x - j_c x = 0 \tag{121}$$

$$2\pi f L - \frac{1}{2\pi f C} = 0 \tag{122}$$

Hence,

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{123}$$

Likewise for circuit (b)

$$j_c b - j_L b = 0 \tag{124}$$

$$2\pi f C - \frac{1}{2\pi f L} = 0 \tag{125}$$

Hence,

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{126}$$

The expressions in equations (123) and (126) are generally used to determine the "resonance frequency" of the circuits.

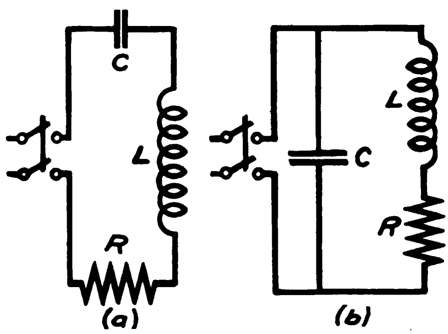

FIG. 59.—Simple series and parallel double energy circuits.

As shown in Chap. VIII a strict application of the definition for resonance gives a different value for the true resonance frequency unless the resistance is negligible. If all the resistance were removed from the circuits in Fig. 59 no energy would be supplied from the bus bars and the stored energy would be transferred back and forth between the inductance and the condensance. With no losses the frequency of the natural or free oscillations would be the same as the "resonance frequency" given in equations (123) and (126).

In circuit a, Fig. 60, and for the oscillograms in Figs. 61 to 65 the condenser is charged from a direct current supply main after which the switch "S" is thrown to the right so as to form an independent closed circuit with the condenser, C, resistance, R, and inductance, L, in series. The energy stored in the condenser is dissipated into heat by the Ri^2 losses during a series of oscillations between the dielectric and magnetic fields.

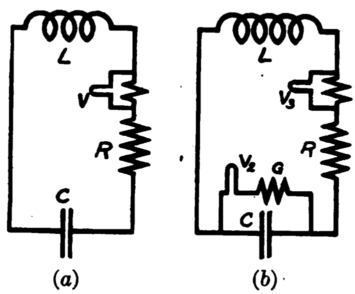

FIG. 60.—Simple oscillatory double energy circuits.

From Kirchoff's Laws the voltage in the closed circuit, Figs. 60 to 64, while the energy originally stored in the condenser is dissipated into heat, is expressed by equations (127) or (128).

$$L\frac{di}{dt} + Ri + \frac{\int idt}{C} = 0 \tag{127}$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0 \tag{128}$$

This is a homogeneous differential equation of the second order and its general solution is given by equation (129), in which A_1 and A_2 are the arbitrary constants.

$$i = A_1\epsilon^{u_1t} + A_2\,\epsilon^{u_2t} \tag{129}$$

In equation (129)

$$u_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \tag{130}$$

$$u_2 = -\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \tag{131}$$

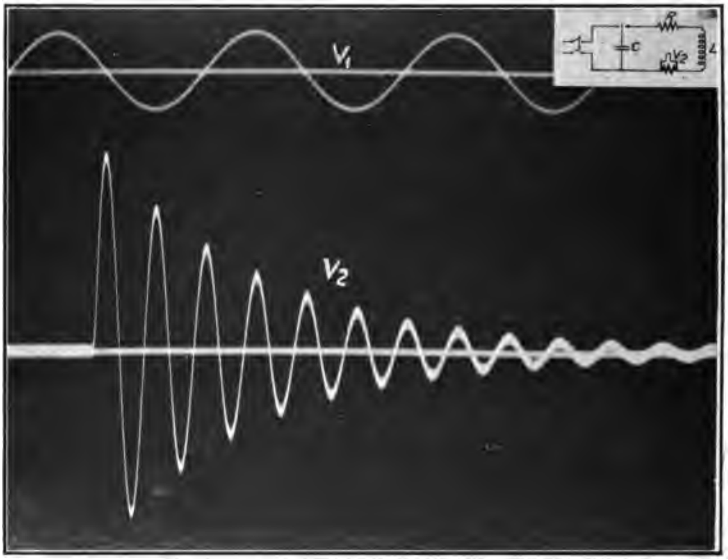

FIG. 61.—Double energy transient.
E = 120 volts; R = 40 ohms; G = 0; L = 0.205 henrys; C = 0.813 microfarads; timing wave 100 cycles.

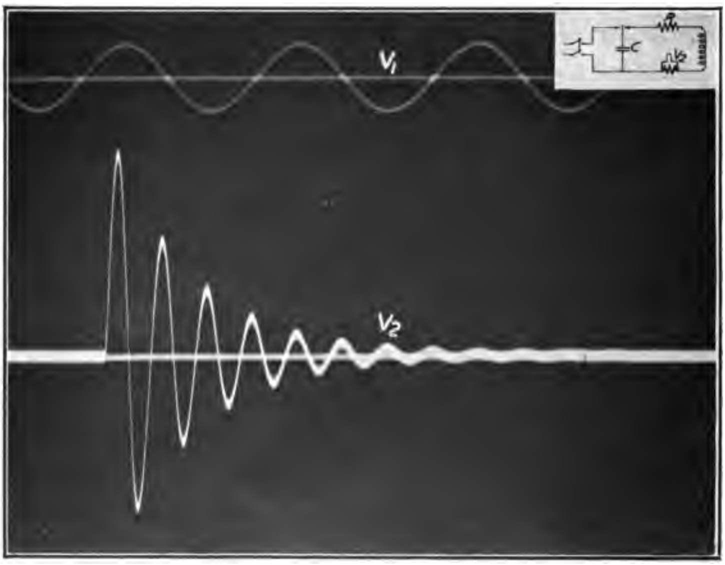

FIG. 62.—Double energy transient.
E = 120 volts; R = 75 ohms; G = 0; L = 0.205 henrys; C = 0.873 microfarads; timing wave 100 cycles.

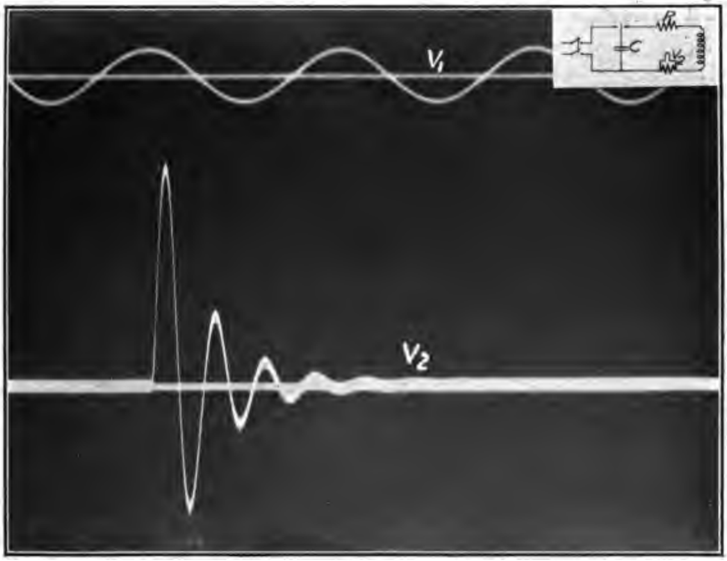

FIG. 63.—Double energy transient.
E = 120 volts; R = 150 ohms; G = 0; L = 0.205 henrys; C = 0.813 microfarads; timing wave 100 cycles.

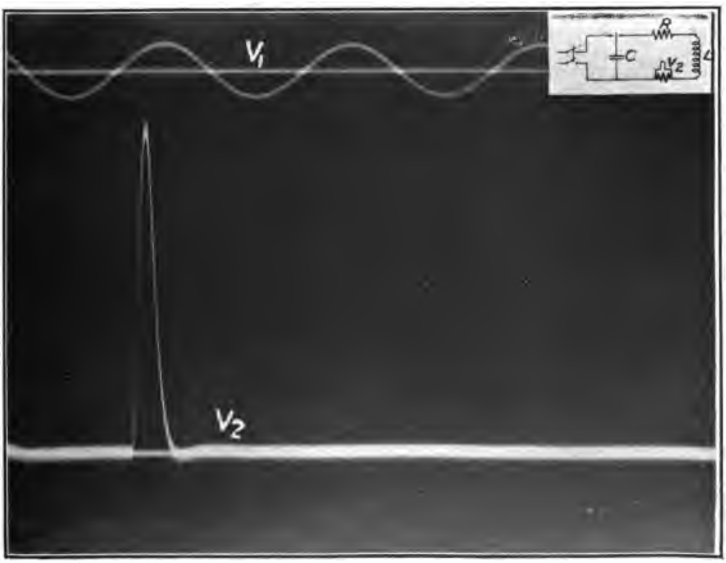

FIG. 64.—Double energy transient.
E = 700 volts; R = 770 ohms; G = 0; L = 0.205 henrys; C = 0.813 microfarads; timing wave 100 cycles

In order to more readily keep the dissipation or damping factors separate from the parts indicating oscillations, equations (130), (131) are rewritten in (132), (133):

$$u_1 = -\frac{R}{2L} + j\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{132}$$

$$u_2 = -\frac{R}{2L} - j\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{133}$$

From (129), (132), (133):

$$i = A_1\epsilon^{-\frac{Rt}{2L}}\epsilon^{j\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t} + A_2\epsilon^{-\frac{Rt}{2L}}\epsilon^{-j\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t} \tag{134}$$

But from Euler's equation for the sine and cosine:

$$\epsilon^{j\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} = \epsilon^{j\omega t} = \cos \omega t + j \sin \omega t \tag{135}$$

$$\epsilon^{-j\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} = \epsilon^{-j\omega t} = \cos \omega t - j \sin \omega t \tag{136}$$

Hence from (134), (135), (136):

$$i = A_1\epsilon^{-\frac{Rt}{2L}}[\cos \omega t + \sin \omega t] + A_2\epsilon^{-\frac{Rt}{2L}}[\cos \omega t - j \sin \omega t] \tag{137}$$

From (135), (136):

$$\omega = 2\pi f = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{138}$$

Hence,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{139}$$

In circuits for which the quantity under the radical is real, oscillations occur at a definite frequency as determined by equation (139) and as illustrated by the oscillograms in Figs. 61 to 63.

If the resistance,

$$R > 2\sqrt{\frac{L}{C}} \tag{140}$$

the quantity under the radical sign in (139) becomes imaginary, and hence the circuit is non-oscillatory. All the energy initially stored in the condenser is dissipated into heat as the voltage and current decrease to zero. This condition is illustrated by the oscillograms in Figs. 64 and 65.

For circuits having comparatively little resistance the natural frequency of oscillations, as given in equation (139), is very nearly the same as the "resonance frequency" given by equations (123) or (126). Thus for the circuit data in Fig. 62 the natural frequency of oscillation, using equation (139), is given in equation (141), while the corresponding "resonance frequency" from equation (126), is given in equation (142).

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = 390 \text{ cycles per second} \tag{141}$$

$$f = \frac{1}{2\pi\sqrt{LC}} = 391 \text{ cycles per second} \tag{142}$$

For circuits corresponding to the conditions that would exist if the condenser in Fig. 60*b* were leaky, similar equations may be obtained. The voltage equation, based on Kirchoff's Laws for circuits of the type shown in Fig. 60*b*, and in Figs. 66 to 70, is expressed by equations (143) and (149).

$$L\frac{di^2}{dt^2} + R\frac{di}{dt} + \frac{{}_ci}{C} = 0 \tag{143}$$

Using the notation shown in the circuit diagram, Fig. 60*b*, letting ${}_ce$ be the voltage across the condenser terminals, and applying Ohm's and Kirchoff's Laws.

$${}_ci = i + {}_Gi \tag{144}$$

$${}_ci = G_Le \tag{145}$$

$${}_ce = Ri + L\frac{di}{dt} \tag{146}$$

Hence,

$${}_ci = i + GRi + GL\frac{di}{dt} \tag{147}$$

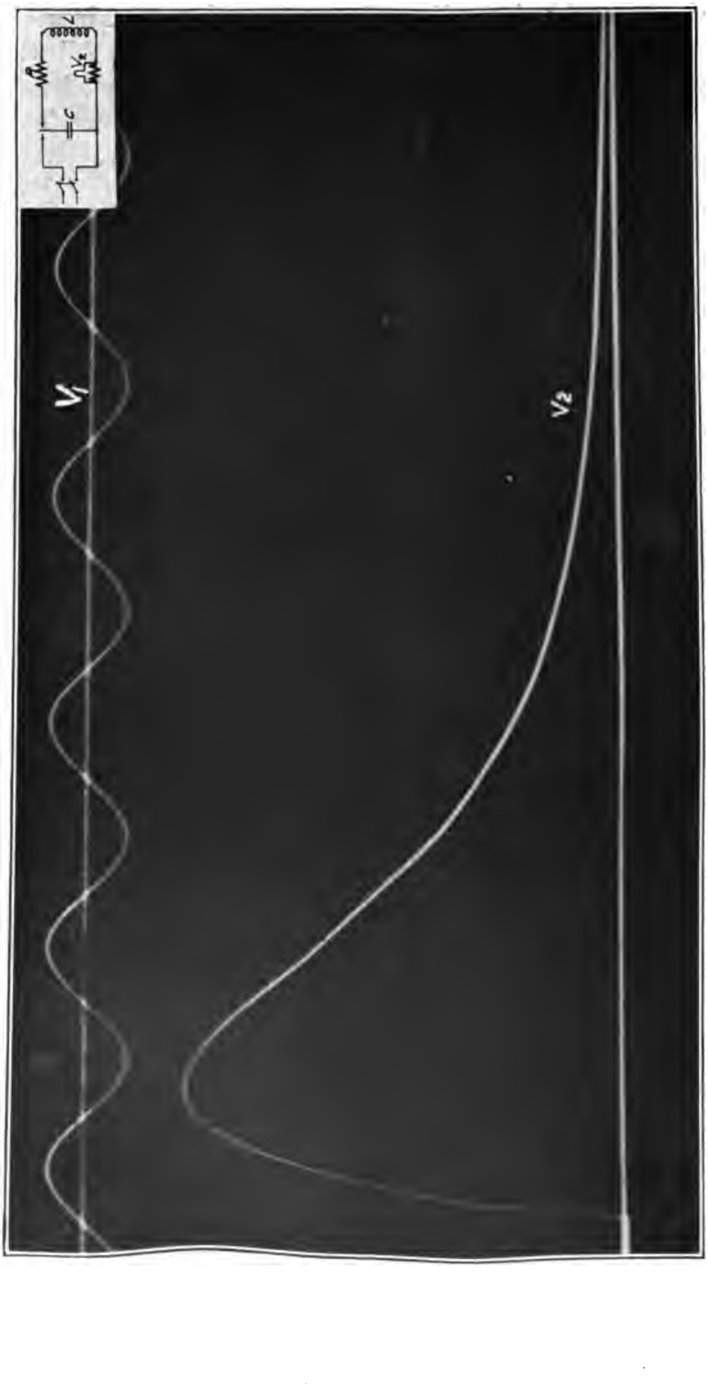

FIG. 65.—Double energy transient. $E = 120$ volts; $R = 70$ ohms; $G = 0$; $L = 0.205$ henrys; $C = 170$ microfarads; timing wave 100 cycles.

From (143) and (147),

$$LC\frac{d^2i}{dt^2} + (RC + GL)\frac{di}{dt} + (1 + GR)i = 0 \qquad (148)$$

or,

$$\frac{d^2i}{dt^2} + \left(\frac{R}{L} + \frac{G}{C}\right)\frac{di}{dt} + \left(\frac{1 + GR}{LC}\right)i = 0 \qquad (149)$$

Equation (149) is a homogeneous differential equation of the second order of the same form as equation (133). Hence, the same general solution applies to both equations, as expressed by equation (150), in which B_1 and B_2 are the two arbitrary constants.

$$i = B_1e^{v_1t} + B_2e^{v_2t} \qquad (150)$$

$$v_1 = -\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) + \sqrt{\frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2 - \frac{1}{LC}} \qquad (151)$$

$$v_2 = -\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) - \sqrt{\frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2 - \frac{1}{LC}} \qquad (152)$$

Rewriting (151), (152) so as to more clearly indicate the damping and oscillation factors, equations (153), (154) are obtained.

$$v_1 = -\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) + j\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} \qquad (153)$$

$$v_2 = -\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) - j\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} \qquad (154)$$

From (150), (153), (154),

$$i = B_1\epsilon^{-\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right)t}\epsilon^{j\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2}t} + B_2\epsilon^{-\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right)t}\epsilon^{-j\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2}t} \qquad (155)$$

From Euler's equation,

$$\epsilon^{j\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2}t} = \epsilon^{j\omega t} = \cos\omega t + j\sin\omega t \qquad (156)$$

$$\epsilon^{-j\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)}t} = \epsilon^{-j\omega t} = \cos\omega t - j\sin\omega t \qquad (157)$$

Hence,

$$i = B_1\epsilon^{-\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right)t}[\cos\omega t + j\sin\omega t] + B_2\epsilon^{-\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right)t}[\cos\omega t - j\sin\omega t] \qquad (158)$$

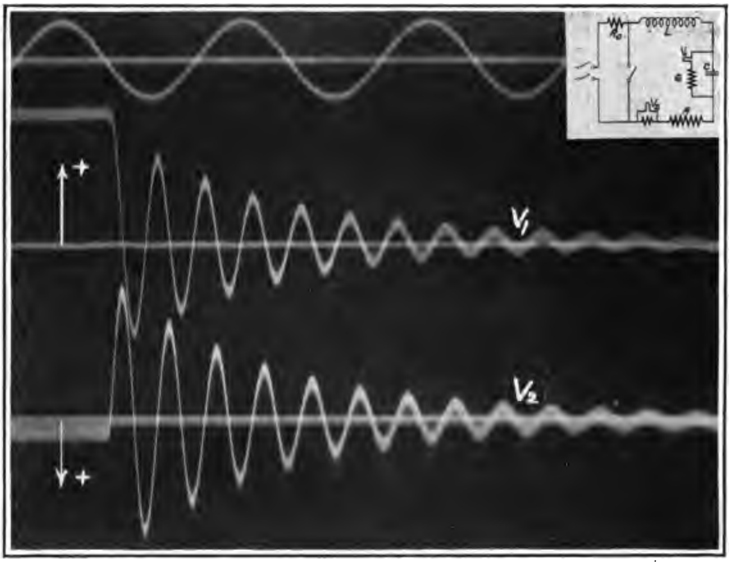

FIG. 66.—Double energy transient.
E = 625 volts; R = 4.5 ohms; G = 1.67 · 10^{-4} mhos.; L = 0.205 henrys; C = 0.813 microfarads; timing wave 100 cycles.

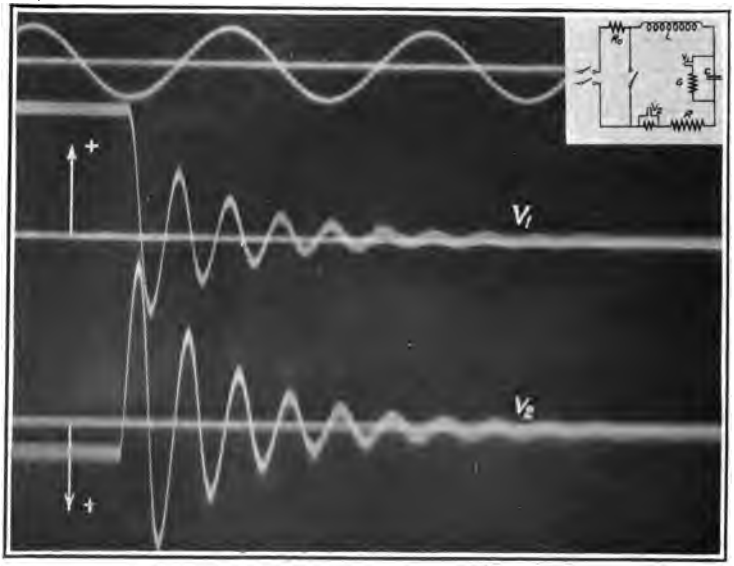

FIG. 67.—Double energy transient.
E_r = 640 volts; R = 4.5 ohms; G = 3.33 · 10^{-4} mhos; L = 0.205 henrys; C = 0.313 microfarads; timing wave 100 cycles.

From (156), (157),

$$\omega = 2\pi f = \sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} \tag{159}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} \tag{160}$$

The circuit is non-oscillatory if

$$\frac{R}{L} - \frac{G}{C} > \frac{2}{\sqrt{LC}} \tag{161}$$

For circuits in which the quantity under the radical sign is greater than zero, the energy in the condenser will be dissipated into heat during a series of oscillations of definite frequency as determined by equation (160) and as illustrated by the oscillograms in Figs. 66, 67 and 68.

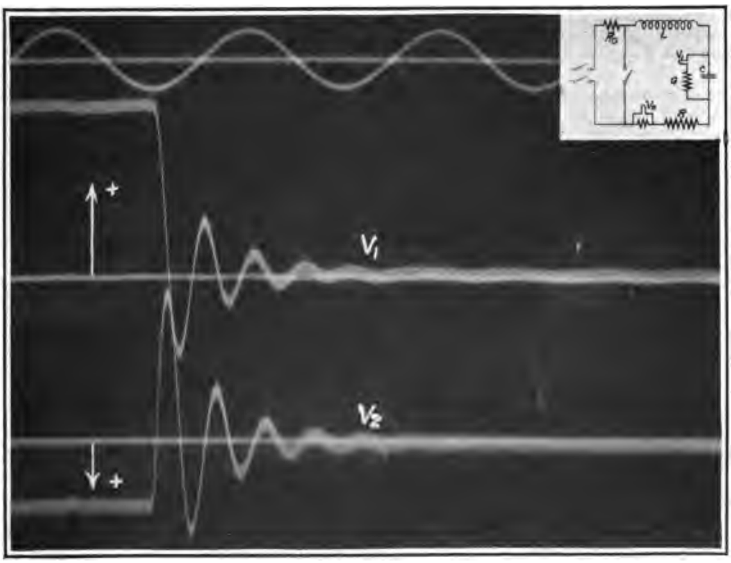

FIG. 68.—Double energy transient.
E = 625 volts; R = 4.5 ohms; $G = 6.66 \cdot 10^{-4}$ mhos; L = 0.205 henrys; C = 0.813 microfarads; timing wave 100 cycles.

If the resistance and the conductance are of such values relatively to the inductance and the condensance that the quantity under the radical sign in (160) becomes imaginary,

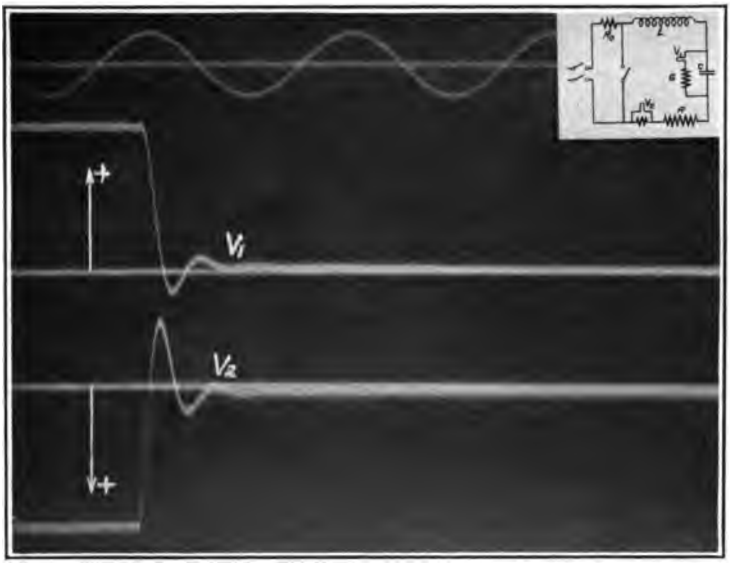

FIG. 69.—Double energy transient.
$E = 550$ volts; $R = 4.5$ ohms; $G = 1.67 \cdot 10^{-3}$ mhos; $L = 0.205$ henrys; $C = 0.813$ microfarads; timing wave 100 cycles.

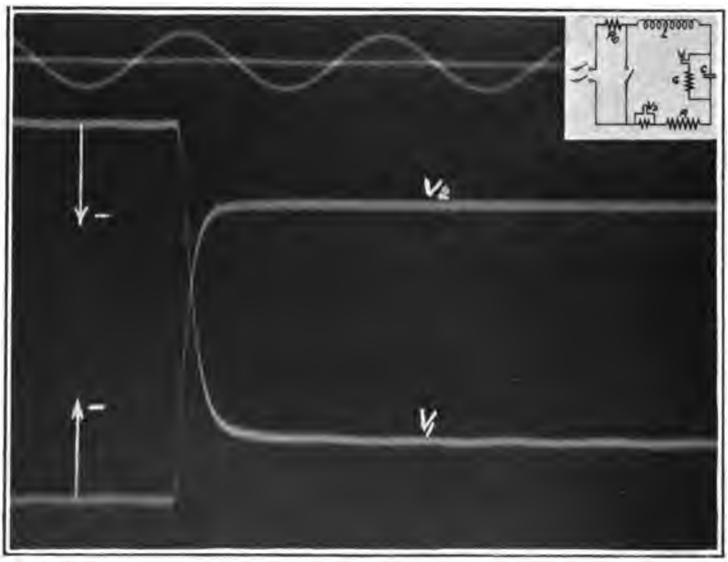

FIG. 70.—Double energy transient.
$E = 550$ volts; $R = 4.5$ ohms; $G = 4.4 \cdot 110^{-3}$ mhos; $L = 0.205$ henrys; $C = 0.813$ microfarads; timing wave 100 cycles.

the circuit would be non-oscillatory. The energy initially stored in the condenser is dissipated into heat while the voltage and current decrease to zero, as illustrated by the oscillograms in Figs. 69 to 70.

The circuits in which the resistance and conductance are comparatively small the natural frequency of oscillation is very nearly the same as the resonance frequency or the natural frequency of circuits in which R and G are equal to zero. Thus for the circuit in Fig. 67 the natural oscillation frequency,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} = 391 \text{ cycles per second} \quad (162)$$

Considering R and G as negligible in determining the frequency of oscillation,

$$f = \frac{1}{2\pi\sqrt{LC}} = 391 \text{ cycles per second} \quad (163)$$

A very interesting circumstance is revealed by equation (160). A circuit having resistance greater than the critical value for oscillatory discharges as given in (161), may be made oscillatory by increasing the conductance across the terminals of the condenser without changing the resistance. This is illustrated by the oscillograms in Figs. 71, 72, 73 and 74.

For the given circuit constants in Fig. 71 the circuit is non-oscillatory. Letting R, L and C remain constant and of the same value as in Fig. 71 but increasing the conductance, G, the circuit is made oscillatory in Fig. 72 although the damping factor is greater than for the circuit in Fig. 71. In Fig. 73 the oscillation was greatly reduced and by still further increasing the conductance while R, L and C remain constant, the circuit is again made non-oscillatory as shown by the oscillogram in Fig. 74.

Dissipation Constant and Damping Factor in Simple Double Energy Circuits.—In the solution for the current in double energy circuits, Fig. 60*a* and Figs. 61 to 65, as given in equation (137), the damping factor and the dissi-

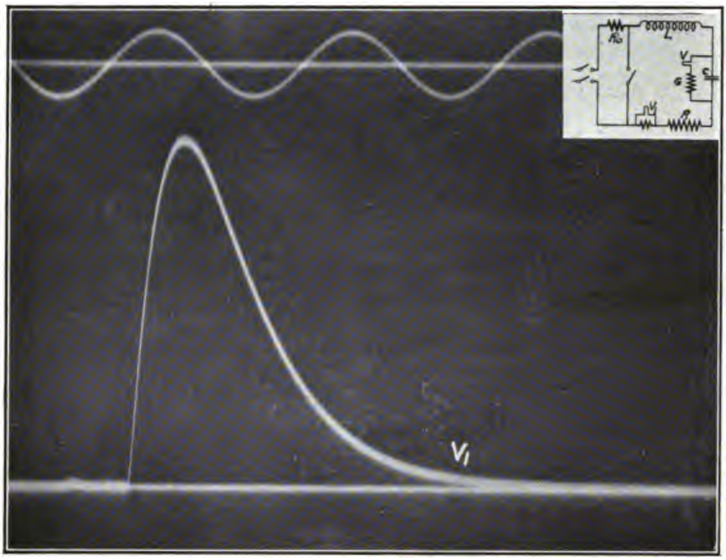

FIG. 71.—Double energy transients.
$E = 700$ volts; $R = 150$ ohms; $G = 0$ mhos; $L = 0.205$ henrys; $C = 36$ microfarads; timing wave 100 cycles.

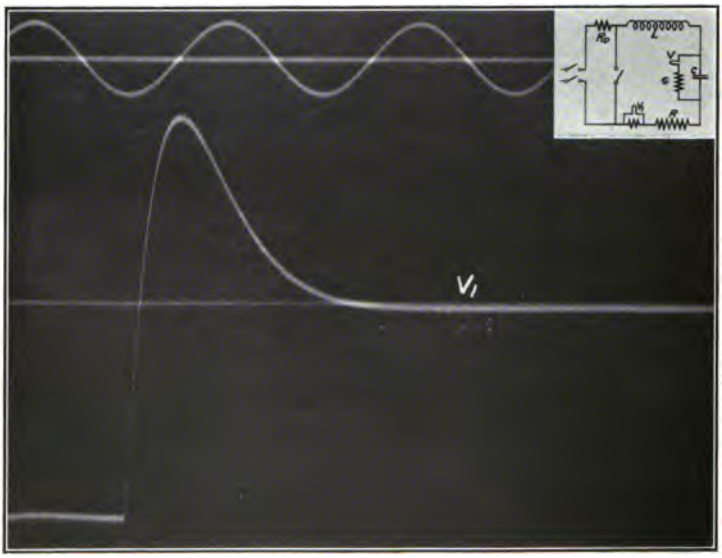

FIG. 72.—Double energy transients.
$E = 700$ volts; $R = 150$ ohms; $G = 4.35 \times 10^{-3}$ mhos; $L = 0.205$ henrys; $C = 36$ microfarads; timing wave 100 cycles.

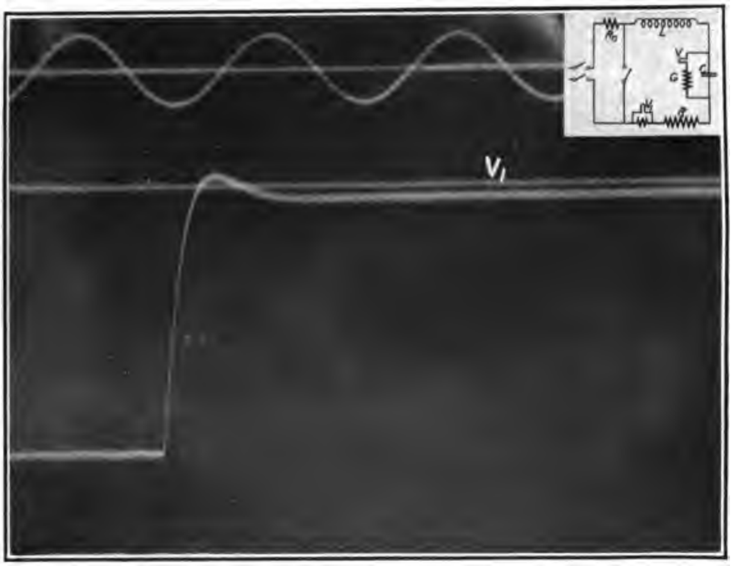

FIG. 73.—Double energy transients.
$E = 400$ volts; $R = 150$ ohms; $G = 1.31 \cdot 10^{-2}$ mhos; $L = 0.205$ henrys; $C = 18$ microfarads; timing wave 100 cycles.

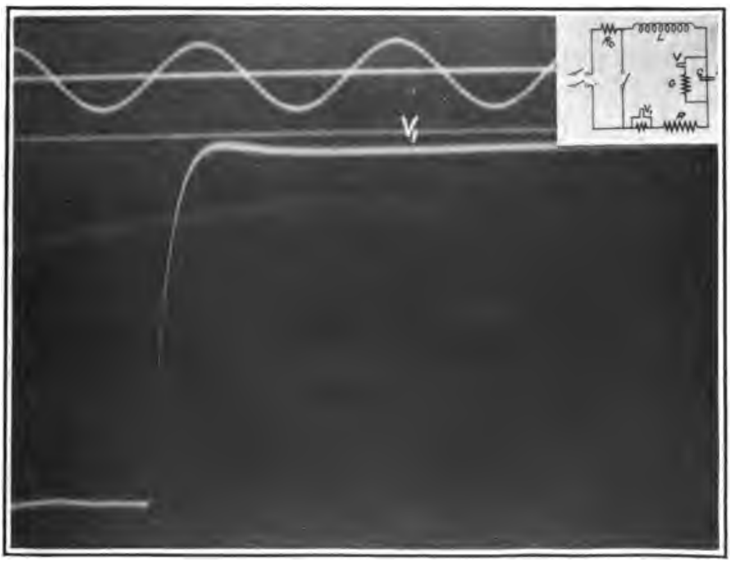

FIG. 74.—Double energy transients.
$E = 700$ volts; $R = 150$ ohms; $G = 2.63 \cdot 10^{-2}$ mhos; $L = 0.205$ henrys; $C = 36$ microfarads; timing wave 100 cycles.

pation constant have already been found. Similarly for circuits in Fig. 60*b* and in Figs. 66 and 67, the factors may be obtained from equation (158)

$$\text{Dissipation or damping constant} = -\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) \quad (164)$$

$$\text{Damping factor} = \epsilon^{-\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right)t} \quad (165)$$

While the above expressions are obtained mathematically by the solution of the differential equation of the circuit, it is important that the student gain a clear concept of the physical phenomena involved.

In Chap. III it was shown that for single energy transient in circuits having resistance and inductance in series the time constant is directly proportional to the inductance and inversely to the resistance.

$$_{L}T_1 = \frac{L}{R} \quad (166)$$

Similarly for circuits having condensance in parallel with conductance, the time constant is directly proportional to the condensance and inversely to the conductance.

$$_{c}T_1 = \frac{C}{G} \quad (167)$$

In double energy circuits the energy is alternately stored in the magnetic and dielectric fields. In circuits having inductance, resistance, condensance and conductance, arranged as shown in the circuit diagrams in Figs. 66 to 70, energy is dissipated into heat both in the resistance and in the conductance. The rate of dissipation is greatest in the conductance, Ge^2, when the voltage across the condenser is a maximum, that is, at the instant all the energy is stored in the dielectric field. Similarly the rate of dissipation in the resistance, Ri^2, is a maximum, when the current is a maximum, that is, when all the energy is stored in the magnetic field. It is evident that since the energy is oscillating it will be in the dielectric field half of the time and in the magnetic field half of the time. Since the energy

is in the dielectric form only half the actual time, the rate of dissipation in the conductance will be equal to half of what would be the case for the same circuit constants in the corresponding single energy transient. Hence, the time constant, ${}_cT_2$, for the dielectric half of the double energy circuit would be twice the time constant, ${}_cT_1$, in the corresponding single energy transient.

$$ {}_cT_2 = 2{}_cT_1 = \frac{2C}{G} \tag{168} $$

Similarly the time constant, ${}_LT_2$, for the inductance-resistance part of the double energy circuit would be twice the time constant, ${}_LT_1$, for the corresponding single energy transient.

$$ {}_LT_2 = 2{}_LT_1 = \frac{2L}{R} \tag{169} $$

Under the given circuit conditions with R, L, G and C constant, the proportionality law applies to double energy transients on the same basis as for single energy transients. The transient term is therefore expressed by the exponential equation and appears as a factor in the current-time and voltage-time equations and represents the dissipation of energy into heat by the resistance, Ri^2, and the conductance, Ce^2, in the circuit.

Let u represent the dissipation constant of double energy circuits. The damping factor is therefore,

$$ \epsilon^{ut} = \epsilon^{-\frac{t}{{}_LT_2}}\epsilon^{-\frac{t}{{}_cT_2}} = \epsilon^{-\frac{R}{2L}t}\epsilon^{-\frac{C}{2G}t} \tag{170} $$

$$ = \epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t} \tag{171} $$

$$ u = -\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) \tag{172} $$

This is the same value as obtained in equation (158).

For circuits in which $G = 0$, as illustrated by Figs. 61 to 65, the term G/C would be zero.

Hence,

$$u' = -\frac{R}{2L} \tag{173}$$

$$\epsilon^{u't} = \epsilon^{-\frac{R}{2L}t} \tag{174}$$

This corresponds to the value of the damping factor in equation (137).

For circuits similar to Figs. 66 to 70 but in which $R = 0$, the term R/L would be zero.

Hence,

$$u'' = -\frac{G}{2C} \tag{175}$$

$$\epsilon^{u''t} = \epsilon^{-\frac{G}{2C}t} \tag{176}$$

As it is not possible to completely eliminate the resistance in circuits having inductance, the conditions for u'' can not be fully realized experimentally.

Equations for Current and Voltage Transients.—For simple double energy circuits, with R, L and C in series, as in Fig. 60a, the general equation (137) for the current is,

$$i = A_1\epsilon^{-\frac{Rt}{2L}}[\cos \omega t + j \sin \omega t] + A_2\epsilon^{-\frac{Rt}{2L}}[\cos \omega t - j \sin \omega t] \tag{177}$$

In equation (177) A_1 and A_2 are the arbitrary constants, which for any specific case are determined by the permanent circuit conditions preceding and following the transient period. Equation (177) may be written in a more compact form as in (178), in which A_3 and A_4 are the arbitrary constants which for any specific case may be determined from the given limiting conditions under which the transient occurred.

$$i = \epsilon^{-\frac{Rt}{2L}}[A_3 \cos \omega t + A_4 \sin \omega t] \tag{178}$$

The voltage across the terminals of the condenser,

$$_ce = -Ri - L\frac{di}{dt}\,. \tag{179}$$

From (178), (179),

$$_ce = \epsilon^{-\frac{Rt}{2L}}\left\{\frac{R}{2}[A_3 \cos \omega t + A_4 \sin \omega t] + \omega L[A_4 \cos \omega t - A_3 \sin \omega t]\right\} \quad (180)$$

For the transients in Figs. 61 to 65 the starting conditions are:

$$t = 0;\ i = 0;\ _ce = E \quad (181)$$

From (178), (180), (181),

$$A_3 = 0;\ A_4 = -\frac{E}{\omega L} \quad (182)$$

Hence,

$$i = -\frac{E}{\omega L}\epsilon^{-\frac{Rt}{2L}} \sin \omega t \quad (183)$$

$$_ce = E\epsilon^{-\frac{Rt}{2L}}[\cos \omega t + \frac{R}{2\omega L} \sin \omega t] \quad (184)$$

For the given circuit constants, $\frac{R}{2\omega L}$ is very small and hence,

$$_ce = E\epsilon^{-\frac{Rt}{2L}} \sin \omega t \text{ (very nearly)} \quad (185)$$

To illustrate the application of equations (183) and (185) for the solution of numerical problems, equations (186) and (187) give the value of the current and voltage in amperes and volts for the oscillograms in Fig. 62.

$$i = -0.24\ \epsilon^{-182.t} \sin (170760°t) \text{ amperes} \quad (186)$$

$$e = 120.\ \epsilon^{-182t} \cos (170760°t) \text{ volts} \quad (187)$$

For the circuit in Fig. 60*b* the equations are of a similar nature. The general equation (158) for the current is given in (188) and may be written in a more compact form as in equation (189), in which B_3 and B_4 are constants that in each case depend on the permanent circuit conditions preceding and following the transient period.

$$i = B_1\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}[\cos \omega t + j \sin \omega t] + B_2\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}[\cos \omega t - j \sin \omega t] \quad (188)$$

$$i = \epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}[B_3 \cos \omega t + B_4 \sin \omega t] \tag{189}$$

$$_ce = -Ri - L\frac{di}{dt} \tag{190}$$

From (188) and (190),

$$_ce = \epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}\left\{\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)[B_3 \cos \omega t + B_4 \sin \omega t] + \omega t[B_4 \cos \omega t - B_3 \sin \omega t]\right\} \tag{191}$$

For the transients in Figs. 66 to 70 the initial conditions are:

$$t = 0; i = 0; {}_ce = E \tag{192}$$

From (189), (191) and (192),

$$B_3 = 0; B_4 = -\frac{E}{\omega L} \tag{193}$$

Hence,

$$i = -\frac{E}{\omega L}\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}\sin \omega t \tag{194}$$

$$_ce = E\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}\left[\cos \omega t + \frac{1}{2\omega}\left(\frac{R}{L}-\frac{G}{C}\right)\sin \omega t\right] \tag{195}$$

If in equation (195),

$$\frac{R}{L} = \frac{G}{C} \tag{196}$$

$$_ce = E\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}\cos \omega t \tag{197}$$

and

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{198}$$

As an illustration of the application of equations (194) and (197) to the solution of a specific problem, let the numerical values of the circuit constants in Fig. 67 be used. Equations (199) and (200) give the instantaneous values of the current and voltage for the oscillogram in Fig. 67.

$$i = -1.28\,\epsilon^{-216.t}\sin (170760°t) \text{ amperes} \tag{199}$$

$$_ce = 640\,\epsilon^{-216.t}\cos (170760°t) \text{ volts} \tag{200}$$

FIG. 75.—Starting current and voltage transients. $E = 125$ volts; $R = 5.0$ ohms; $G = 0.00083$ mhos; $L = 0.205$ henrys; $C = 9.0$ microfarads; timing wave 100 cycles; $f = 117$ cycles.

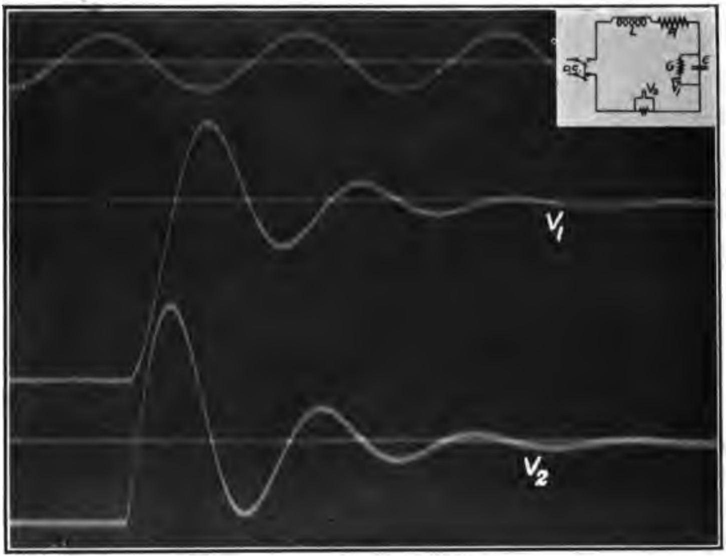

FIG. 76.—Starting current and voltage transients.
E = 125 volts; R = 5.0 ohms; G = 0.00167 mhos; L = 0.205 henrys; C = 9.0 microfarads; timing wave 100 cycles.

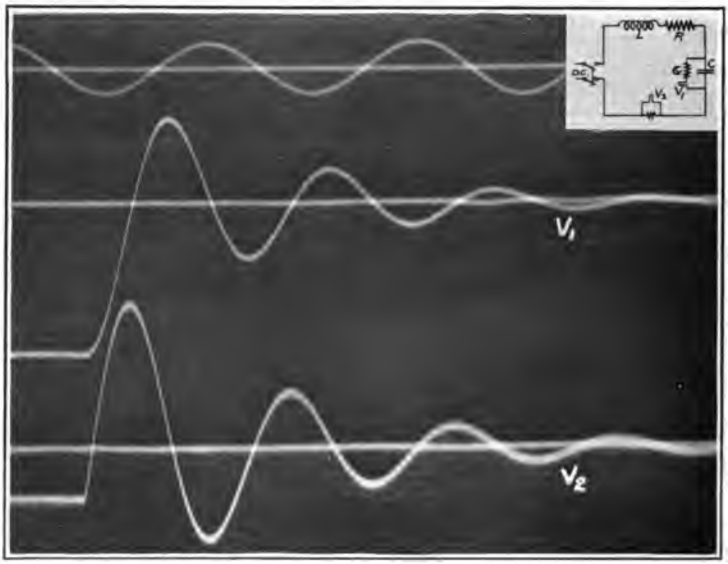

FIG. 77.—Starting current and voltage transients.
E = 125 volts; R = 5.0 ohms; G = 0.0025 mhos; L = 0.205 henrys; C = 9.0 microfarads; timing wave 100 cycles.

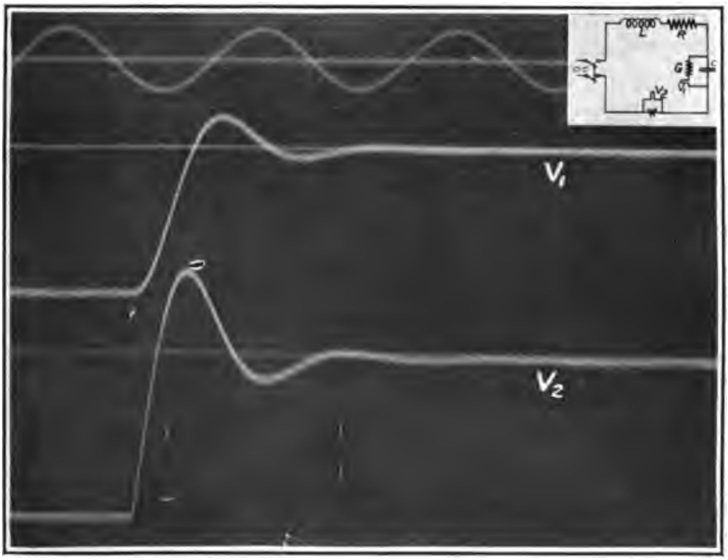

FIG. 78.—Starting current and voltage transients.
$E = 125$ volts; $R = 5.0$ ohms; $G = 0.005$ mhos; $L = 0.205$ henrys; $C = 9.0$ microfarads; timing wave 100 cycles.

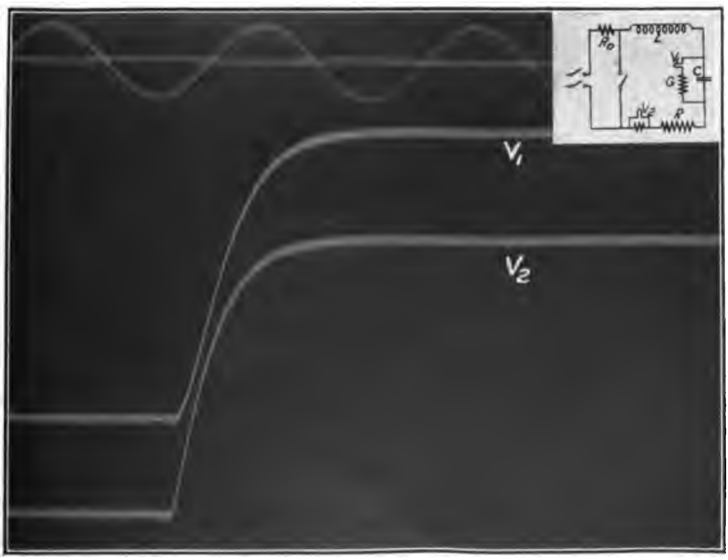

FIG. 79.—Starting current and voltage transients.
$E = 125$ volts; $R = 5.0$ ohms; $G = 0.0132$ mhos; $L = 0.205$ henrys; $C = 9.0$ microfarads; timing wave 100 cycles.

The oscillogram in Fig. 74 shows the current and voltage transients in a circuit having a high damping factor but in which the frequency of oscillation is the same as if both R and G were zero. The data in Fig. 74 show that the circuit constants were of such values as to satisfy equation (196).

For the oscillograms in Figs. 75 to 79 the circuits are of the same type as in Figs. 66 to 70, but the permanent conditions preceding and following the transition period are different. The oscillograms show the starting current and voltage transients at the points in the circuit indicated by the positions of the vibrators in the circuit diagram and for the values of R, L, G and C, as given in each figure.

Problems and Experiments

1. Given a circuit similar to Fig. 60(a) having R, L, and C in series. Let $R = 20$ ohms, $L = 0.31$ henrys, $C = 1.2$ microfarads and $E = 120$ volts, the initial condenser discharge voltage.

(a) Find the natural period of oscillation of the circuit.

(b) Find the time constant, and the damping factors.

(c) Write the equation for the transient condenser discharge current.

(d) For what values of R would the circuit be non-oscillatory.

2. Derive the equations for ${}_ce$, the transient voltage across the condenser terminals in Fig. 62. Trace the voltage-time curve for ${}_ce$ on rectangular coordinates, using the same time scale on the X axis as in the oscillogram.

3. Take a double energy oscillogram similar to Fig. 58. Obtain all the necessary data and write the equations for the transient current.

4. Write the equations for the voltage and current curves of the oscillogram in Fig. 61 similar to equations (199) and (200) for Fig. 67 in the text.

5. Take a series of oscillograms similar to Figs. 66 to 70. Find the values of the circuit constants and place on the film ampere and volt scales for the current and voltage curves.

6. For the oscillogram in Fig. 75 with the given circuit conditions:

(a) Write the expression for ${}_ce$ and i similar to equations (194), (195).

(b) Insert the numerical values of circuit constants and express ${}_ce$ and i in volts and amperes, similar to equations (199) and (200).

CHAPTER VI

ELECTRIC LINE OSCILLATIONS, SURGES AND TRAVELING WAVES

Electric lines whether designed for power transmission or telephone service, may be considered as consisting of an infinite series of infinitesimal double energy circuits of the simple types discussed in Chap. V. Each infinitesimal length of line may be represented by the resistance and inductance in one of the series circuit elements in Fig. 80 and the corresponding portion of the dielectric between the conductor and neutral by the conductance and condensance in the adjacent parallel circuit. The line constants, R, L, G and C, depend on the size and spacing of the conductors and the electrical properties of the dielectric and conductor materials. To readily gain clear concepts of transmission line phenomena it is essential for the student to conduct experiments and obtain quantitative test data. Commercial transmission lines are seldom available for experimental work but artificial lines having the electrical characteristics of actual lines can be readily constructed of convenient design for operation in the laboratory.

Artificial Electric Lines.—Since the operating characteristics of transmission lines are determined by the line constants, the resistance, inductance, conductance and condensance and are independent of the space and mass factors, much of the experimental work can to good advantage be performed on equivalent artificial electric lines.[1] The oscillograms of electric line transients used for illustrations in this chapter were obtained from an artificial transmission line,[2] one section of which is shown in Fig.

[1] Dr. A. E. Kennelly, "Artificial Transmission Lines."

[2] *Trans.* A. I. E. E., vol. 31, p. 1137.

81. This line is of the lumpy "T" type of design, which means that each unit has resistance and inductance in series combined with condensance and conductance in parallel as shown in Fig. 82. If the insulation is sufficiently high the conductance factor may be omitted and the section circuit diagram would be as in Fig. 83, which represents the circuit diagram for the "T" unit in Fig. 81.

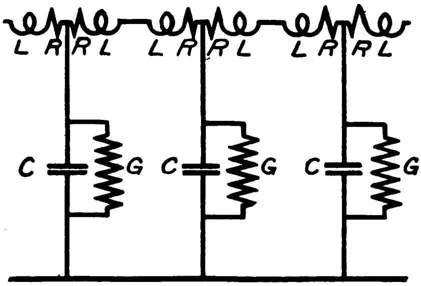

FIG. 80.—Transmission line circuit diagram showing three elements.

In the lumpy types the line constants R, L, G and C, are massed instead of uniformly distributed as in actual lines. As the lumpy type only approximates a uniform distribution of the resistance, inductance, conductance and condensance in the line, the size of each unit must be small in comparison to the total length of the line. In Fig. 81 is shown one of the twenty ten-mile units in the artificial transmission line in the electrical engineering laboratories of the University of Washington. In each unit the line constants may be adjusted within the following limits:

Resistance, minimum value, 2.59 ohms.
Inductance, maximum value, 0.021 henry.
Condensance, 0.1 to 1.0 microfarad.

The resistance may be increased to any desired amount by moving the clamp on the resistance loop or by inserting resistance elements between the units; the inductance may be decreased by turning the right hand coil and by taps in the lower coil; and the condensance may be varied in steps by using ten or a less number of condensers in

series. Adjustments can be made so as to give to the artificial line the electrical constants equivalent to an actual transmission line of any size of wire up to No. 0000 A.W.G. hard-drawn copper and for any spacing up to 120 inches.

FIG. 81.—Section of artificial electric line, University of Washington.

The line may also be adjusted so as to be equivalent to commercial telephone lines.

Time, Space and Phase Angles.—In Chap. V the equations for the current and voltage transients were derived for

simple double energy circuits, Fig. 60, in which the circuit constants, R, L, G and C are massed. Evidently the energy transfer between the magnetic and dielectric fields would be of essentially the same nature if the inductance and resistance were intermixed with the condensance and conductance or uniformly distributed as in a transmission line. However, one important difference must be noted which necessitates an additional factor in the expression for the transient current and voltage. In circuits having massed circuit constants the maximum value of the voltage

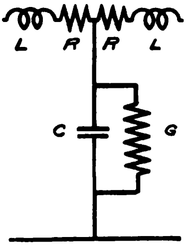

FIG. 82.—T-circuit with leaky condenser.

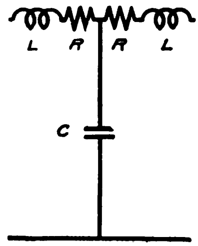

FIG. 83.—T-circuit.

will be impressed on all of the condensance at the same instant, and all parts of the magnetic field reach a maximum at the instant the current is a maximum. On the other hand, with R, L, G and C distributed, as in long transmission lines, the time required for the electric wave to travel along the length of the line enters into the problem. If a constant direct current voltage is impressed at one end of an electric line a short but definite time will elapse before the voltage reaches the other end of the line. If an alternating current is transmitted over the line the successive waves travel over the line at definite velocity in the same manner as the impulse from the direct current voltage. The maximum point of any wave travels at a definite velocity as determined by the distribution of the resistance, inductance, conductance and condensance in the line. In transmission lines with air as the dielectric and with copper or aluminum conductors the speed at which a wave or impulse

travels is approximately the same as the velocity of propagation of an electromagnetic wave in space or the velocity of light.

$$v = 3 \cdot 10^{10} \text{ cm. per second} \tag{205}$$

In a medium having a permeability μ and a permittivity k,

$$v' = \frac{3 \cdot 10^{10}}{\sqrt{\mu\kappa}} \text{ cm. per second} \tag{206}$$

The time required for the voltage wave to travel a distance x along the line having distributed line constants, depends on the distance and velocity of propagation.

$$t_1 = \frac{x}{v} \tag{207}$$

In comparing the transient voltage and current conditions at any two points on an electric line, x distance apart, consideration must be given to the time required for the propagation of the electric wave over the given distance and hence the factor t, must be included in the equations. In double energy circuits having massed R, L, G and C, as in the oscillograms in Figs. 66 to 69, and for oscillations produced by the discharge of energy initially stored in the condensers, the instantaneous values of the voltage and current, under the stated conditions, are given in equations (194), (197). Under similar conditions, as illustrated by the oscillograms in Figs. 84 to 91, and by the introduction of space angles, the equations may be considered as applying to circuits having distributed R, L, G and C, as in transmission lines.

To simplify the notations, let

$$u = \frac{1}{2}\left(\frac{R}{L} - \frac{G}{C}\right) \tag{208}$$

$$I = -\frac{E}{\omega L} \tag{209}$$

$$\gamma = \text{time phase angle for } t = 0 \tag{210}$$

For oscillations in circuits with massed R, L, G and C, under the stated assumptions:

$$i = I\epsilon^{-ut} \sin (\omega t - \gamma) \tag{211}$$
$$e = E\epsilon^{-ut} \cos (\omega t - \gamma) \tag{212}$$

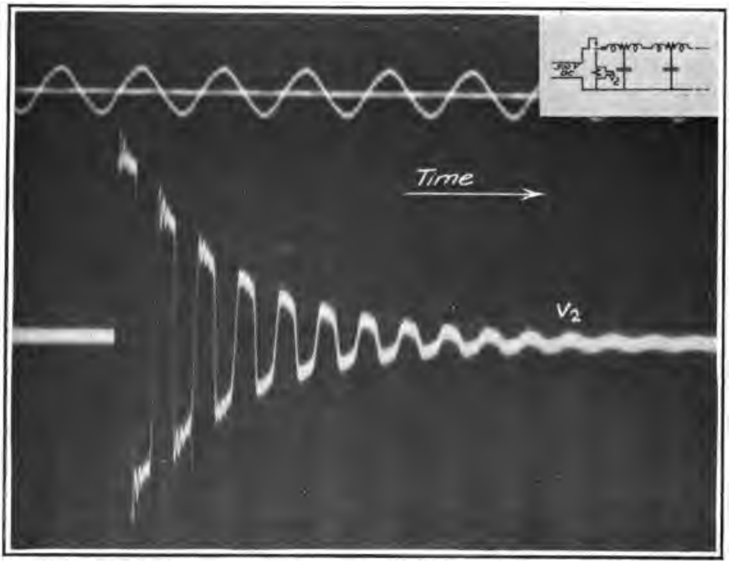

FIG. 84.—Electric line oscillations.
E = 500 volts; R = 52.14 ohms; G = 0; L = 0.427 henrys; C = 3.66 microfarads; length = 232 miles; timing wave 100 cycles.

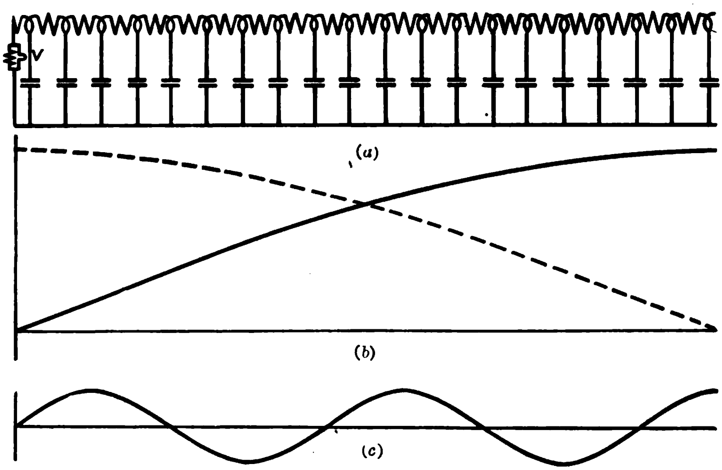

FIG. 85.—Circuit and wave diagram for Fig. 84.

For oscillations in circuit with distributed R, L, G and C, under similar conditions:

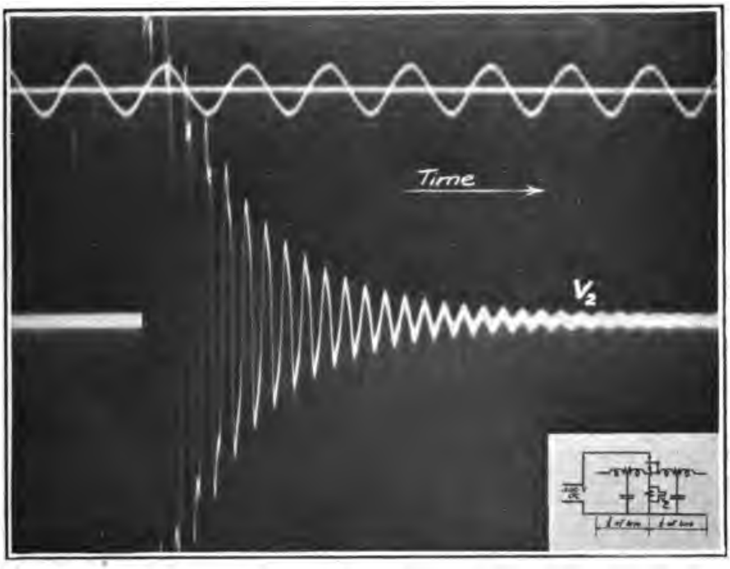

FIG. 86.—Electric line oscillations.
$E = 500$ volts; $R = 26.12$ ohms; $R_2 = 26.02$ ohms; $G_1 = 0$; $G_2 = 0$; $L_1 = 0.2128$ henrys; $L_2 = 0.2146$ henrys; $C_1 = 1.831$ microfarads; $C_2 = 1.834$ microfarads; timing wave 100 cycles.

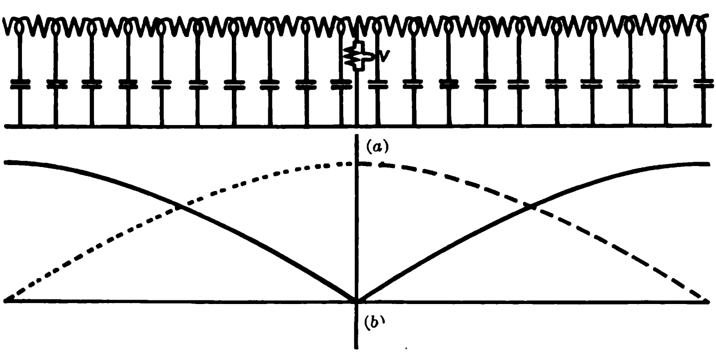

FIG. 87.—Circuit and wave diagram for Fig. 86.

$$i = I\epsilon^{-ut} \sin [\omega(t - t_1) - \gamma] \tag{213}$$
$$e = E\epsilon^{-ut} \cos [\omega(t - t_1) - \gamma] \tag{214}$$

Substituting ϕx for ωt_1:

$$i = I\epsilon^{-ut} \sin (\omega t - \phi x - \gamma) \tag{215}$$

$$e = E\epsilon^{-ut} \cos (\omega t - \phi x - \gamma) \tag{216}$$

In equations (215), (216) ωt is the time angle, ϕx the space angle and γ the phase angle.

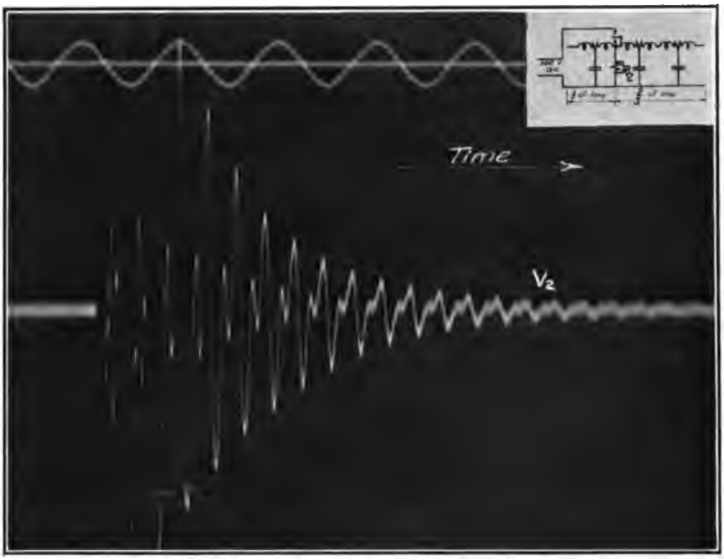

FIG. 88.—Electric line oscillations.

E = 500 volts; R_1 = 31.28 ohms; R_2 = 15.64 ohms; G_1 = 0; G_2 = 0; L_1 = 0.2564 henrys; L_2 = 0.1282 henrys; C_1 = 2.201 microfarads; C_2 = 1.10 microfarads; timing wave 100 cycles.

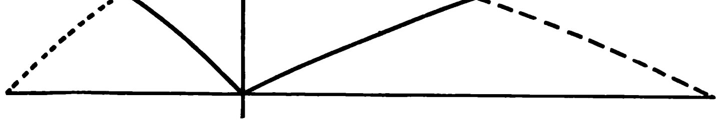

FIG. 89.—Circuit and wave diagram for Fig. 88.

Natural Period of Oscillation.—Since the space angle, ϕx, in equations (215), (216) is directly proportional to

the distance, x, from the origin, it is evident that the phase of the current, i, and the voltage, e, changes progressively along the line. At some distance, l_0, the current and volt-

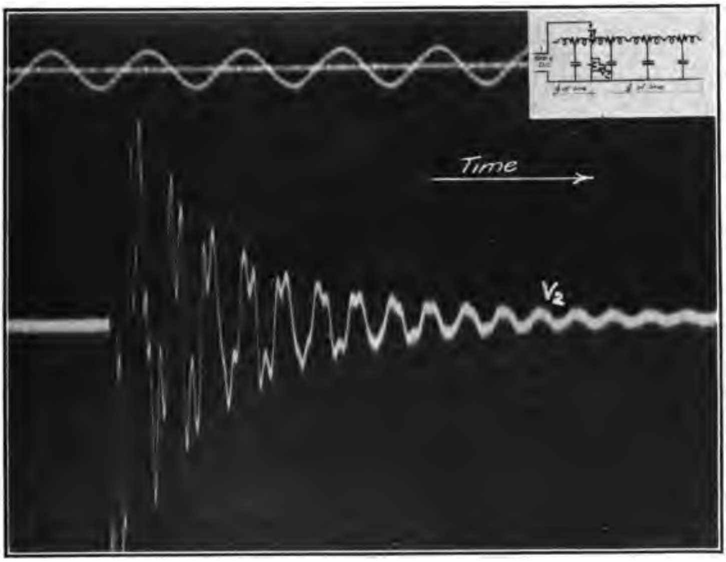

FIG. 90.—Electric line oscillations.
E = 500 volts; R_1 = 39.10 ohms; R_2 = 13.04 ohms; G_1 = 0; G_2 = 0; L_1 = 0.3204 henrys; L_2 = 0.1070 henrys; C_1 = 2.748 microfarads; C_2 = 0.917 microfarads; timing wave 100 cycles.

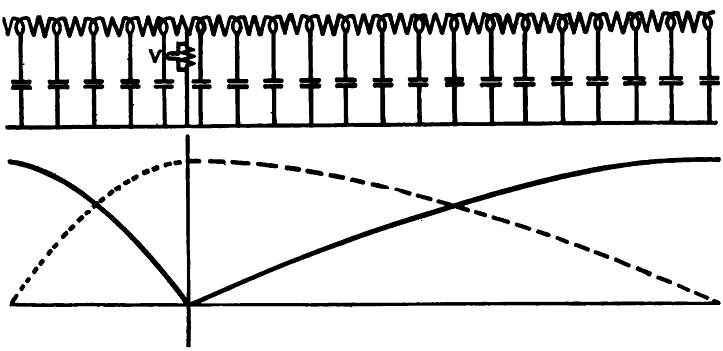

FIG. 91.—Circuit and wave diagram for Fig. 90.

age are displaced 360 deg. from their starting point values. The distance, l_0, is called the wave length and is the distance

the electric field travels during the time, t_0, required for the completion of one cycle or complete wave.

If f is the frequency of oscillations in cycles per second,

$$t_0 = \frac{1}{f} \text{ seconds} \tag{217}$$

$$l_0 = vt_0 \tag{218}$$

The fundamental frequency or natural period of free oscillation depends on the length of the line and on the imposed circuit conditions. For the oscillations recorded in the oscillogram in Fig. 84, the line is open at the receiver end and connected through the vibrator circuit at the generator end. The diagram in Fig. 85*b* shows that under these conditions the length of the line is one-fourth wave length of the fundamental oscillations. In Fig. 85*c* is shown the wave diagram for the ninth harmonic which appears as ripples on the fundamental oscillation.

In Fig. 86 the vibrator is connected at the middle point leaving both ends open. The corresponding wave diagram in Fig. 87*b* shows that the length of the line is two quarter-wave lengths or one-half wave length, and the frequency of the fundamental oscillation is twice that in Fig. 84. Similarly in Fig. 88, in which the vibrator is connected at one-third the distance from one end, each of the two parts becomes a vibrating element giving fundamental oscillations. The frequency of the oscillation of the shorter part is twice as great as for the longer portion. In Fig. 90, with the vibrator connected at one-fourth the distance from one end of the line, the short end oscillates at three times the frequency of the long end. In all cases the voltage and current vary progressively along the line so that at any instant the average voltage, instead of the maximum value, is impressed on the condensers and the average current, in place of the maximum value, flows through the inductance.

The same results would be obtained in circuits with massed R, L, G and C in which the maximum voltage is

impressed on all the condensance simultaneously or all of the magnetic field reaches a maximum at the instant the current is a maximum, by reducing the condensance and inductance in the ratio of the maximum to the average values. This ratio is $\pi/2$ for sine waves.

The frequency for free oscillations in simple circuits with *massed* R, L, G and C was derived in Chap. V, equation (162).

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} \text{ cycles per second} \qquad (219)$$

The frequency of free oscillations in circuits having *distributed* R, L, G and C and a sine wave distribution of the voltage and current may be obtained by multiplying L and C in equation (219) by $\pi/2$, the ratio of the maximum to the average value.

$$f = \frac{1}{4}\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} \text{ cycles per second} \qquad (220)$$

In commercial electric lines the quantity $\frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2$ is negligibly small in comparison with $1/LC$. For practical problems the frequency of the fundamental oscillations or surges in transmission lines with uniformly distributed R, L, G and C may therefore be obtained by equation (221).

$$f = \frac{1}{4\sqrt{LC}} \text{ cycles per second} \qquad (221)$$

Thus the fundamental frequency of oscillation for the transmission line in Fig. 84,

$$\text{Thus; } f = \frac{1}{4\sqrt{0.427 \cdot 3.665 \cdot 10^{-6}}} = 200.0 \text{ cycles per second} \qquad (222)$$

This may be checked by measurements on the oscillogram in Fig. 84. On the original film (the cut in the text is reduced in size) 10 cycles of the timing wave measured 14.3 cm., while 10 cycles of the transient oscillations measured 7.1 cm. Hence the frequency,

$$f = \frac{143.}{0.71} = 200.1 \text{ cycles per second} \qquad (223)$$

Since L and C represent the total inductance and condensance of the line the frequency depends on the total length of the line or the length of time in which the oscillation occurs, as illustrated by the oscillogram in Figs. 84, 86, 88 and 90. The transmission line, or other circuits of distributed R, L, G and C, therefore, have a fundamental frequency at which the whole line oscillates, but as any fractional part of the line may also oscillate independently of the whole line, particularly if the oscillating section is short as compared to the entire line, oscillations of any frequency may occur. At high frequencies the successive waves are so close together that a small variation in the time constants will cause them to overlap. Since R, L, G and C are not perfectly constant high frequency oscillations interfere with each other, and on this account resonance phenomena occur only at low or moderate frequencies.

Length of Line.—In ordinary transmission lines, with air as the dielectric and conductors of copper or aluminum, an electric wave or impulse travels approximately $3 \cdot 10^{10}$ cm. per second, the velocity of propagation of an electromagnetic field in free space, equation (205). This fact is of much practical importance in transmission line calculations. If the length of the line is known the frequency of the fundamental oscillation and of the harmonics can readily be determined. The length of the line is one quarter wave length of the fundamental oscillation as illustrated by the oscillogram in Fig. 84 and corresponding diagrams in Fig. 85.

$$v = 4f_0 l_0 \tag{224}$$

Conversely, if the frequency of the oscillation is known the length of the oscillating section may be determined. In artificial transmission lines with the frequency of the fundamental oscillation obtained from oscillograms the equivalent length of the line can be calculated. Thus from measurements on the oscillogram in Fig. 84, equation (223), $f = 200$ cycles per second. Hence the length of the line,

$$l_0 = \frac{v}{4f_0} = \frac{3 \cdot 10^{10}}{4.200} \text{ cm.} = 375 \text{ km.} = 233 \text{ miles} \quad (225)$$

From equations (205), (221) and (224), relations are obtained by which L or C may be calculated if the length of the line, l in cm., and either C or L are known.

$$v = 3 \cdot 10^{10} = 4fl = \frac{l}{\sqrt{LC}} \quad (226)$$

Hence,

$$L = \frac{l^2}{v^2 C} \text{ or } C = \frac{l^2}{v^2 L} \quad (227)$$

For cables or circuits in which the permeability, μ, and the permittivity, κ, are greater than unity the corresponding relations are obtained from equations (206), (221) and (224).

$$v' = \frac{3 \cdot 10^{10}}{\sqrt{\mu \kappa}} = 4fl = \frac{l}{\sqrt{LC}} \quad (228)$$

$$L = \frac{l^2}{v'^2 C} \text{ or } C = \frac{l^2}{v'^2 L} \quad (229)$$

These equations are useful in the calculation of the condensance of circuits in which the inductance can be more easily determined, as in complex overhead systems and in calculating inductance in cables or other circuits in which the condensance may readily be measured.

Velocity Unit of Length. Surge Impedance.—In handbooks and tables the values of R, L, G and C are given for some unit of length as cm., km., 1,000 ft., mile, etc. In discussions and calculations of transient phenomena the velocity unit of length is sometimes used. For overhead structures the unit of length, l, on this basis would be v, or $3 \cdot 10^{10}$ cm. Hence from equation (227), and under the assumptions made in its derivation,

$$L_v = \frac{1}{C_v} \quad (230)$$

The natural or surge impedance from equations (121), (230):

$$z_v = \sqrt{\frac{L_v}{C_v}} = L_v = \frac{1}{C_v} \tag{231}$$

$$y_v = \sqrt{\frac{C_v}{L_v}} = C_v = \frac{1}{L_v} \tag{232}$$

By the use of the velocity unit of length investigations on transmission systems having sections of different constants and hence of different wave length are greatly simplified. In systems having overhead lines, cables, coiled windings, as in transformers, arresters, etc., the wave length becomes the same in the velocity measure of length.

Voltage and Current Oscillations and Power Surges.—It has been shown that in free or stationary oscillation transmission lines or other electric circuits having uniformly distributed R, L, G and C the current and voltage are essentially in time quadrature. From equations (215), (216):

$$i = I\epsilon^{-ut} \sin (\omega t - \phi x - \gamma) \tag{234}$$

$$e = E\epsilon^{-ut} \cos (\omega t - \phi x - \gamma) \tag{235}$$

Hence, the instantaneous power, p, at any point in the circuit is given by equation (236):

$$p = ei = \frac{EI}{2} \epsilon^{-ut} \sin 2(\omega t - \phi x - \gamma) \tag{236}$$

The direction of the flow of power changes $4f$ times each second since the sine function becomes alternately positive and negative for successive π time degrees. That is, a surge of power occurs in the circuit of double the frequency of the current or voltage oscillations, although the average flow of power along the line is zero.

$$\text{Average power, } p_0 = 0 \tag{237}$$

General Transmission Line Equations.—In the preceding paragraphs various phases of the electric transients that occur during the free or natural oscillations of electric circuits have been discussed. The general problem, in which transient phenomena occur while continuous power is supplied to the system and transmitted along the line, is

necessarily much more complex. In transmission lines or other electric circuits having uniformly distributed resistance, inductance, conductance and condensance, with R, L, G and C the constants per unit length of line, the voltage and current relations in time may be expressed by partial differential equations as in (238), (239):

$$\frac{\partial e}{\partial x} = L\frac{\partial i}{\partial t} + Ri \tag{238}$$

$$\frac{\partial i}{\partial x} = C\frac{\partial e}{\partial t} + Ge \tag{239}$$

Differentiating (238) with respect to x and (239) with respect to t and eliminating $\frac{\partial^2 i}{\partial x \partial t}$ equations (240), (241) may be derived:

$$\frac{\partial^2 e}{\partial t^2} = LC\frac{\partial^2 e}{\partial t^2} + (RC + GL)\frac{\partial e}{\partial t} + RGe \tag{240}$$

$$\frac{\partial i^2}{\partial t^2} = LC\frac{\partial^2 i}{\partial t^2} + (RC + GL)\frac{\partial i}{\partial t} + RGi \tag{241}$$

A general solution for these equations is given in equation (242), one term of which represents the sum of the outgoing and the other the sum of the incoming waves.

$$e = A_1\epsilon^{\pm at}\,\epsilon^{bx}\sin(\alpha t + \beta x + \gamma_1) + A_2\epsilon^{\pm at}\epsilon^{-bx}\sin(\alpha t + \beta x + \gamma_2) \tag{242}$$

In order to determine the values of A_1, A_2, a, b, α, β, γ_1, and γ_2, the specific conditions under which the line operates must be given. It is, however, of first importance to understand the purpose or functions of each term in the equation. On the basis of energy flow and dissipation in a line transmitting power the following interpretation of the symbols in equation (242) may be helpful.

A_1, and A_2 are constants whose values are determined by the limiting conditions of each specific problem.

ϵ^{-at} may be called the *time damping factor* and a the *time dissipation* constant for the transient oscillations.

This factor represents the same form of energy dissipation as ϵ^{-ut} in Chap. V. Ordinarily the transformation of electric energy into heat by the Ri^2 and Ge^2 losses is non-reversible and therefore the sign of the dissipation constant must be negative.

$\epsilon^{\pm bx}$ may be called the *distance damping factor* and b the *distance dissipation constant.* It relates both to the losses along the line in the steady flow of energy, as in transmission lines carrying permanent load, and to the flow of transient energy in the system as with traveling waves or in the oscillations of compound circuits.

at is the *time angle.* Under permanent or steady conditions with a simple sine voltage, $E \sin \omega t$, impressed at the generating station $a = \omega$ and has only one value. However, if the impressed voltage is a complex wave or during transition periods between two permanent conditions while transient currents and voltages are flowing in the system, a would have more than one value.

βx is the *space or distance angle* with x as the distance along the line from the origin. If waves of more than one frequency are passing over the line β would have more than one value.

γ_1 and γ_2 are phase angles for $t = 0$.

Traveling Waves.—Traveling waves are in many respects similar to free oscillations or standing waves as the transfer of energy between the dielectric and magnetic fields is the basis for all double energy electric phenomena. The essential difference is that in traveling waves power flows along the line while in free oscillations or standing waves the energy oscillates between the two fields but does not travel from one line element to another. Oscillograms of the current and voltage factors in traveling waves are shown in Figs. 92 to 97. It should be noted that the current is in time phase with the voltage for the outgoing waves and differs by 180 deg. for the returning waves. In both cases a flow of power occurs along the line.

In Fig. 92 the receiver end of the line is short circuited. The reflected voltage wave is in opposite time phase to the outgoing wave while the corresponding current waves are in the same direction.

In Fig. 93 the receiver end of the line is open and as a consequence the reflected current wave reverses in sign while the corresponding voltage wave is in the same direction as the outgoing wave.

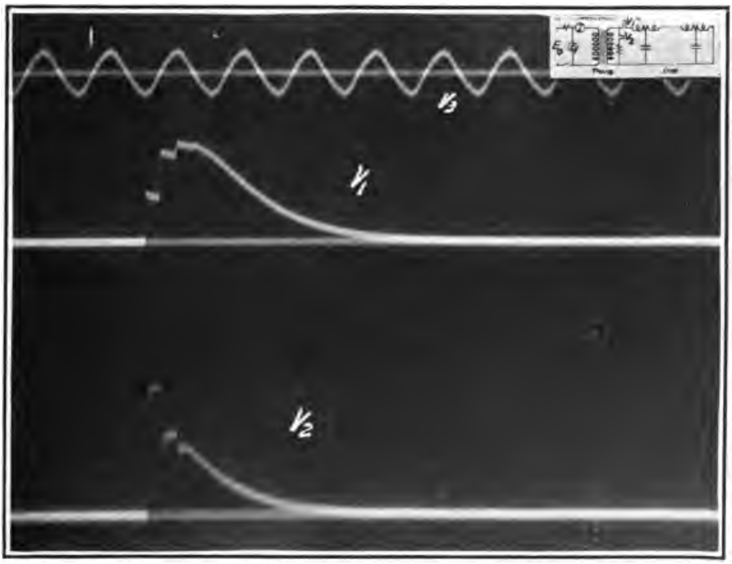

Fig. 92.—Traveling waves on artificial transmission line. Receiver end short circuited.
E_0 = 120 volts, d.c.; E_1 = 5 volts; I_1 = 19.5 amps.; R = 56.1 ohms; G = 0; L = 0.418 henrys; C = 3.053 microfarads; timing wave 100 cycles.

For the circuit in Fig. 94 a resistance equal to the surge impedance of the circuit, $\sqrt{L}/\sqrt{C}$, is inserted at the receiver end of the line. All the energy of the traveling wave was dissipated into heat by the Ri^2 losses at the receiver end of the line and as a consequence there was no reflected voltage or current waves or return flow of power. From the timing wave and known length of line it is found that the velocity of propagation of the impulse is equal to

v or $3 \cdot 10^{10}$ cm. per second, the velocity of propagation of an electromagnetic field in free space.

A traveling wave in an electric line is sometimes transformed into a standing wave, as illustrated by the oscillograms in Figs. 95, 96 and 97. In Fig. 95, with the receiver end of the line open, both the voltage and current waves show that the traveling wave passed from the generator to the receiver end of the line and back again four times before it was changed into a standing wave. During this period the voltage and current waves are in phase or

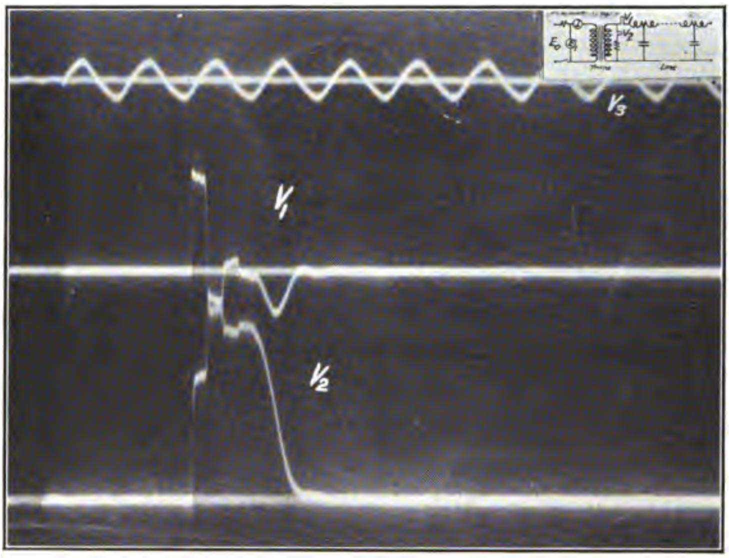

FIG. 93.—Traveling waves on artificial transmission line. Receiver end open. E_0 = 120 volts d.c.; E_1 = 5 volts; I_1 = 19.5 amps. R = 56.1 ohms; G = 0; L = 0.418 henrys; C = 3.053 microfarads; timing wave 100 cycles.

180 deg. apart, showing a flow of power along the line, but when the traveling wave is changed to an oscillation the current leads the voltage (note position of vibrators in the circuit diagram) by 90 deg. If the current leads or lags 90 deg. with respect to the voltage, the power in the circuit is reactive and therefore the average flow of power along the line is equal to zero.

Similarly, the oscillograms in Figs. 96 and 97 show impulses which after passing over the lines several times as traveling waves are transformed into standing waves or oscillations. In each case the impulse starts as a traveling wave with the current and voltage in phase and a flow of power along the line. The oscillogram shows that the traveling wave was converted into an oscillation or standing wave, in which the current and voltage differ by 90 deg. in time phase, in less than one hundredth of a second, and that the energy then oscillated between the magnetic and dielectric fields without flow of power along the line.

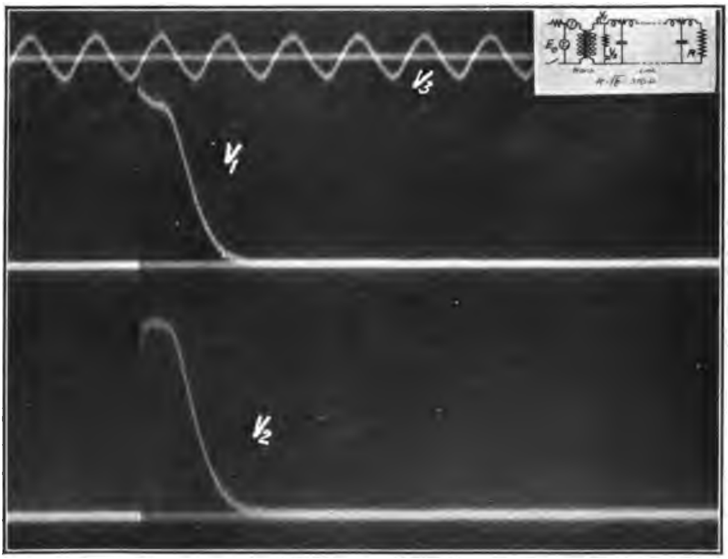

FIG. 94.—Traveling waves on artificial transmission line. Receiver resistance $= \sqrt{L}/\sqrt{C}$; E_0 = 120 volts d. c.; E_1 = 5 volts; I_1 = 19.5 amps.; R = 56.1 ohms; $G = 0$; L = 0.418 henrys; C = 3.053 microfarads; timing wave 100 cycles.

In Fig. 97 the vibrator connections for the voltage wave, v_3, were reversed; the voltage and current were in phase instead of 180° apart as indicated by the oscillogram.

The change in frequency when the traveling wave is converted into a standing wave should be noted. In the

traveling wave the inductance and condensance of the line alone determines the velocity of propagation while for the oscillations or standing waves the line and transformer oscillate together as a compound circuit.

In determining the instantaneous values for the current and voltage at any point on the system the power flow must be taken into consideration in addition to the dissipation of the transient electric energy into heat as expressed by the damping factor ϵ^{-ut}. It is evident that the flow of power may be increasing, decreasing or unvarying in the direction of propagation.

If the power flow is uniform the expressions for the current and voltage are in the simplest form (244), (255), as the power transfer factor does not appear in the equations.

$$i = I_0\epsilon^{-ut} \cos (\omega t \mp \phi x - \gamma) \tag{244}$$

$$e = E_0\epsilon^{-ut} \cos (\omega t \mp \phi x - \gamma) \tag{245}$$

$$p = E_0 I_0\epsilon^{-ut}[1 - \sin^2 (\omega t \mp \phi x - \gamma)] \tag{246}$$

$$\text{Average power, } p = \frac{E_0 I_0}{2}\epsilon^{-2ut} \tag{247}$$

Uniform flow of transient power is infrequent but may occur in special cases. Thus if a transformer line and load, as in Fig. 100, are disconnected from the power supply and left to die down together, uniform flow of power in the line may be realized provided the dissipation constant of the line is equal to the average dissipation constant of the whole system. Consider the transformer as having stored in the magnetic field a comparatively large quantity of energy while its resistance and conductance are relatively small compared to the corresponding value for the line. Likewise assume that the load part of the circuit has very little energy stored in its magnetic and dielectric fields and that its dissipation constant is large as compared to that of the line. Under these conditions the dissipation of energy is most rapid in the load part of the circuit and slowest in the transformer. Hence a flow of energy will occur from the transformer to the load. If the rate of

energy dissipation of the line is midway between the corresponding rates for the load and transformers the energy dissipated in the line would be equal to the amount initially stored in the line while part of the energy originally stored in the transformer flows through the line and is dissipated in the load part of the circuit. The flow of power in the line would be uniform as it delivers to the load part of the circuit all the energy received from the transformer.

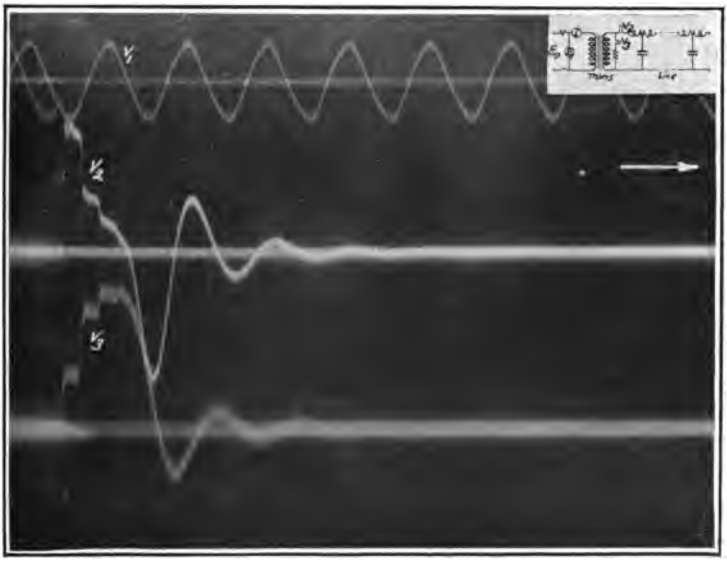

Fig. 95.—Traveling waves changing to standing waves on artificial transmission line.

R = 55.32 ohms; G = 0; L = 0.419 henrys; C = 3.05 microfarads; Length = 207 miles; 4/0 copper; 96 in spacing; Transformer L = 37.8 henrys; timing wave 100 cycles.

The *flow of power decreases* along the line in the direction of propagation, if energy is left in the circuit elements as the traveling wave passes along the line. That is, the traveling wave scatters part of its energy along its path and thus decreases in intensity with the distance traveled. This decrease is expressed by a *power transfer constant, s,* comparable to the power dissipation constant *u.* If no energy were supplied to the line by the traveling wave the

voltage and current would decrease by the dissipation factor ϵ^{-ut}. With power supplied by the flow of energy the decrease would be slower and would be expressed by a combination of the damping and power transfer factors.

For decreasing flow of power:

$$\text{Damping factor} = \epsilon^{-ut} \tag{248}$$

$$\text{Power transfer factor} = \epsilon^{+st} \tag{249}$$

Combined damping and power transfer factor

$$= \epsilon^{-(r-s)t} \tag{250}$$

Similarly if the *flow of power increases* along the line in the direction of propagation the traveling wave receives

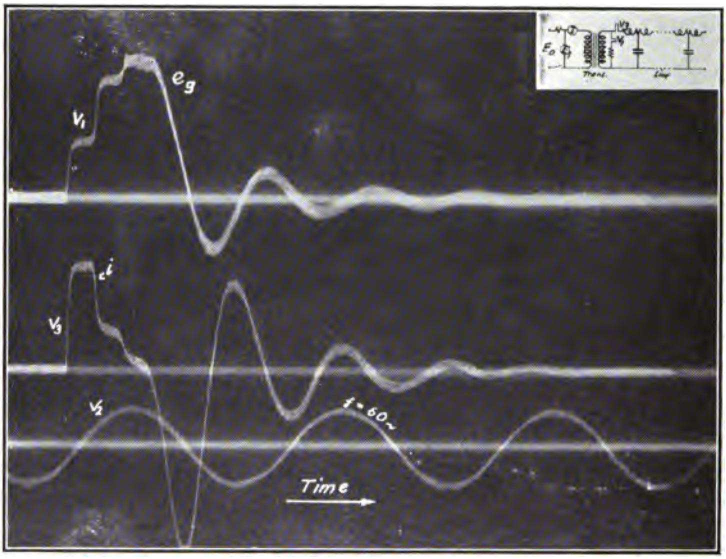

FIG. 96.—Traveling waves changing to standing waves of artificial transmission line.

$E_0 = 110$ volts; $I_1 = 19.8$ amps.; $R = 52.9$ ohms; $G = 0$; $L = 0.412$ henrys; $C = 3.03$ microfarads; timing wave 60 cycles.

energy from the line elements and the actual decrease in the voltage and current is greater than indicated by the dissipation constant. The power transfer would in this case be negative, and the combined damping and power transfer factor would be expressed by equation (253).

For increasing flow of power:

$$\text{Damping factor} = \epsilon^{-ut} \tag{251}$$

$$\text{Power transfer factor} = \epsilon^{-st} \tag{252}$$

Combined damping and power transfer factor

$$= \epsilon^{-(u+s)t} \tag{253}$$

To express the instantaneous values of the current and voltage at any point in the circuit a distance factor must be included. For if the traveling wave either scatters or gathers in energy as it travels along the line the voltage and current factors decrease at a lesser or greater rate, as the case may be, in the direction of propagation than if the flow of power were uniform. In order to use only one power transfer constant, s, in the equation, let λ = the distance x expressed in velocity measure (254)

For decreasing flow of power along the line:

$$\text{the distance damping factor} = \epsilon^{-s\lambda} \tag{255}$$

For increasing flow of power along the line:

$$\text{the distance damping factor} = \epsilon^{s\lambda} \tag{256}$$

The instantaneous values of the transient current, voltage and power under conditions producing a flow of power along the line from the point of reference, in the direction of propagation may be expressed by equations (257), (258), or (259), (260).

$$i = I_o\epsilon^{-(u \mp s)t}\epsilon^{\mp s\lambda}\cos(\omega t \mp \phi\lambda - \gamma) \tag{257}$$

$$e = E_o\epsilon^{-(u \mp s)t}\epsilon^{\mp s\lambda}\cos(\omega t \mp \phi\lambda - \gamma) \tag{258}$$

$$i = I_o\epsilon^{-ut}\epsilon^{\pm s(t-\lambda)}\cos(\omega t \mp \phi\lambda - \gamma) \tag{259}$$

$$e = E_o\epsilon^{-ut}\epsilon^{\pm s(t-\lambda)}\cos(\omega t \mp \phi\lambda - \gamma) \tag{260}$$

$$p = I_oE_0\epsilon^{-2ut}\epsilon^{\pm 2s(t-\lambda)}[1 - \sin^2(\omega t \mp \phi\lambda - \gamma)] \tag{261}$$

$$\text{Average power, } p = \frac{I_0E_0}{2}\epsilon^{-2ut}\epsilon^{\pm 2s(t-\lambda)} \tag{262}$$

The upper sign of $\phi\lambda$ applies to waves traveling in the direction of increasing values of λ and the lower sign for returning waves, for which λ is decreasing. For $s = 0$

which represents a constant flow of power, equations (259) and (260) become identical with equations (244) and (245). Referring to Fig. 100, already used for illustrating the flow of constant power, it is evident that if the dissipation constant for the line is less than the average dissipation constant for the system the flow of power from the transformer will be such as to increase the power stored in the line, while if the line dissipation constant is greater than the average the reverse would be the case.

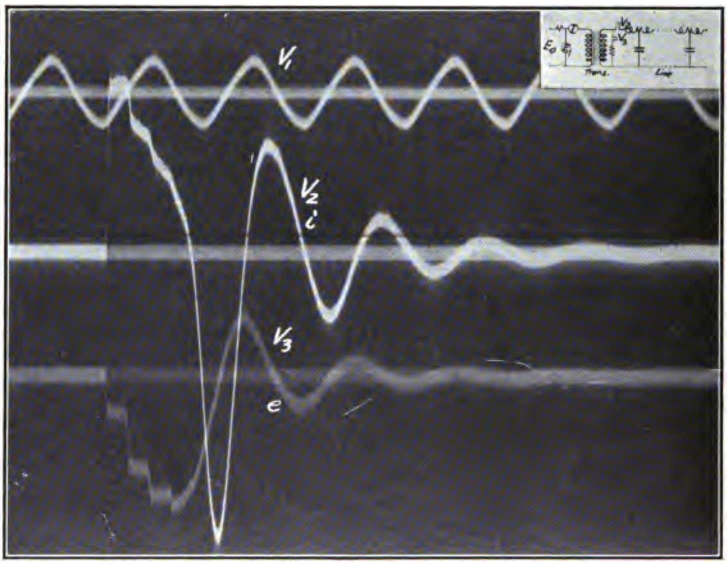

FIG. 97.—Traveling waves changing to standing waves on artificial transmission line.
E_0 = 120 volts; Length = 200 miles; 4/0 copper; 120 in. spacing; timing wave 100 cycles.

Traveling waves are of very frequent occurrence in electric power systems. Not merely such violent disturbances as direct strokes of lightning or short circuits, but practically every change in load or circuit conditions produce transient waves that travel over the system. Simple traveling waves as illustrated by the oscillograms in Figs. 92 to 101 are frequently called *impulses*. In the first part of the

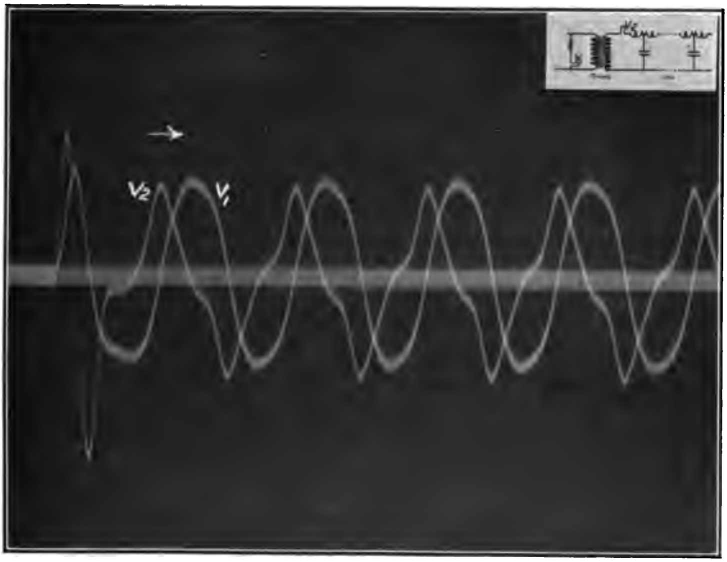

FIG. 98.—Oscillation of compound circuit. Starting transient of artificial transmission line and step-up transformer.

Length of line = 52 miles; 4/0 copper; 96 in. spacing, R = 13.84 ohms ; G = 0; L = 0.105 henrys; C = 0.764 microfarads; transformer L = 37.8 henrys; 60 cycle supply.

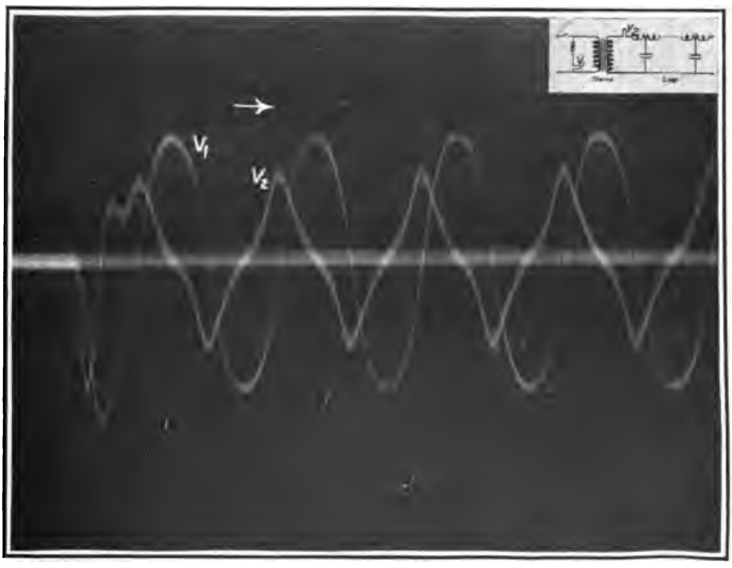

FIG. 99.—Oscillation of compound circuit. Starting transient of (artificial transmssion) line and transformers.

Length of line 52 miles; 4/0 copper; 96 in. spacing; R = 13.84 ohms; G = 0; L = 0.105 henrys; C = 0.764 microfarads; 60 cycle supply.

impulse as it passes along a line the wave energy increases at a rate depending on the steepness of the wave front, and after the maximum value is reached the wave energy decreases. While the wave energy increases the combined dissipation and power transfer factor is represented by $\epsilon^{-(u+s)t}$ as in equation (253), and during the decreasing stage by $\epsilon^{-(u-s)t}$ as in equation (250). The steepness of the wave front which corresponds to the sharpness or suddenness of a blow is often a more important factor in causing damage to the electric system than the quantity of energy involved.

Compound Circuits.—In commercial systems the transmission line is not an independent unit but merely a link between the generator and load circuits. Step-up and step-down transformers, generators and load circuits, lightning arresters and regulating devices, and all the apparatus necessary for the operation of the system are electrically interconnected into one unit. In the several parts of the system the circuit constants differ in relative magnitude and hence the velocity of propagation of an electric impulse varies and no two sections may have the same natural period of oscillation. While the whole system may oscillate as a unit partial oscillations are of much more frequent occurrence.

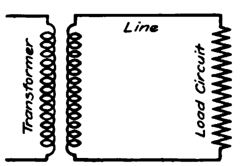

FIG. 100.—Circuit diagram of a compound circuit.

In Figs. 98, 99, 101 and 102 are shown the oscillations of compound circuits consisting of an artificial transmission line and transformers. The ripples on the current wave, v_1, indicate a wave traveling over the transmission line alone. From measurements on the film, Fig. 101, the length of the line is found to be 207 miles. The line and transformers oscillate as a compound circuit at a frequency of 10.5 cycles per second. In Fig. 102 the length of the second half wave is longer than for the first half wave. This is due to a variation in the permeability of the iron in the transformer core.

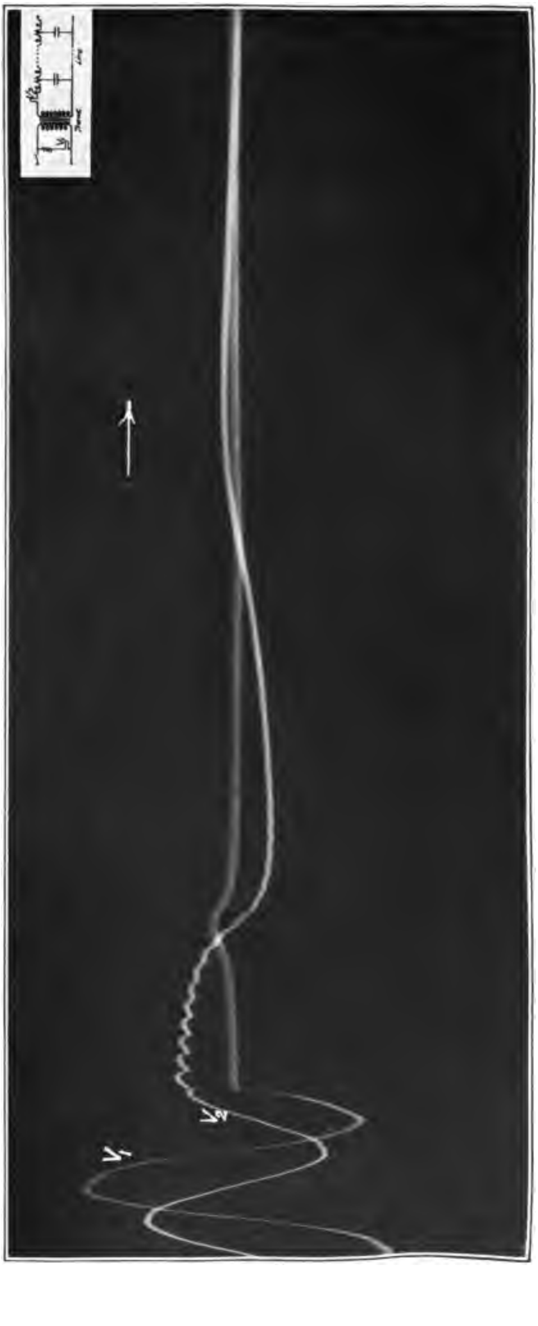

FIG. 101.—Oscillation of a compound circuit. Artificial transmission line and step-up transformer. Length of line = 207 miles; 4/0 copper; 96 in. spacing; 60 cycle supply; oscillation frequency = 10.5 cycles.

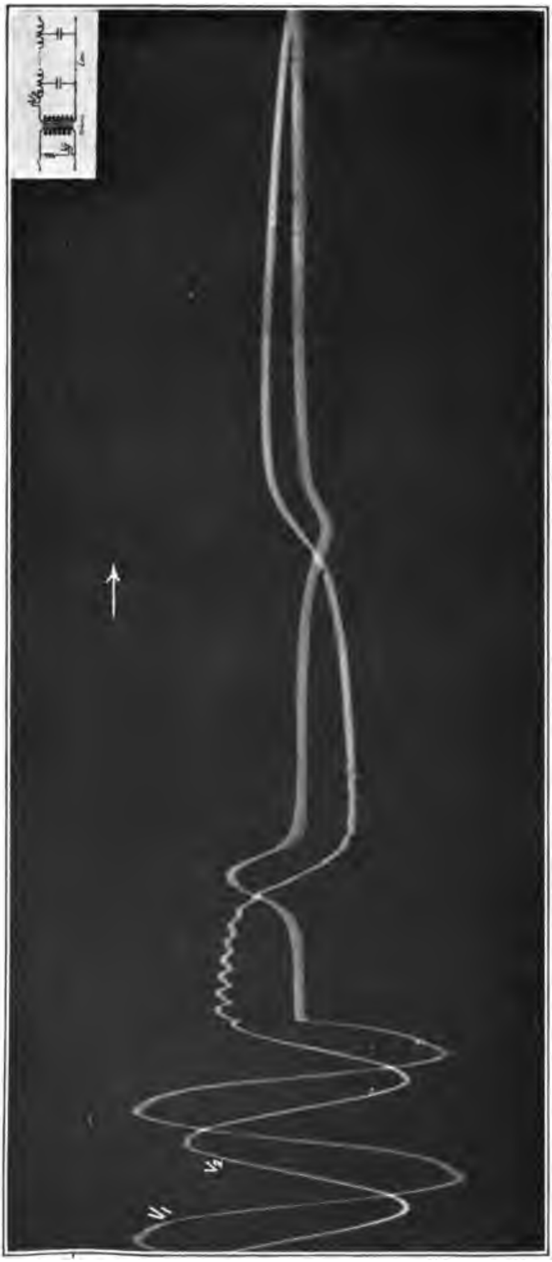

FIG. 102.—Oscillation of a compound circuit. Artificial transmission line and transformers. Length of line = 207 miles; 4/0 copper; 96 in. spacing; 60 cycle supply; oscillation frequency = 11.4 cycles.

Problems and Experiments

1. Given a transmission line, 80 miles long, of No. 0000 copper, spaced 12 ft. and with the receiver end open. From handbook tables obtain the line constants. Find the fundamental oscillation frequency of the line. Check the results by solving for the frequency from the known velocity of propagation of an electric wave in space and use the given length of the line.

2. Make a series of oscillograms similar to Figs. 84, 86, 88 and 90, on an artificial transmission line. From the oscillograms determine the equivalent length of actual line. Check by determining the natural frequency of oscillation from the line constants.

3. From the oscillogram in Fig. 95 or 97 determine the frequency of oscillation of the transmission line alone and the transmission line combined with the transformer. Assume the condensance of the transformer equal to zero. From the data given calculate the inductance of the transformer.

4. In the oscillogram in Fig. 101 the ripples on the voltage wave indicate reflections of traveling waves in the transmission line with the receiver end open. Calculate the length of the line.

5. From the oscillograms and data in Fig. 101 calculate the inductance in the transformer in the compound circuit. Assume the condensance of the transformer equal to zero. It should be noted that the inductance is essentially massed while the condensance is distributed and hence for the combined circuit $f = \dfrac{1}{2\sqrt{2\pi LC}}$.

6. From the data given in Fig. 102 calculate the average inductance of the transformers during the first half cycle after the current and voltage wave lines cross; also during the second half cycle.

7. Make oscillograms of the oscillations of compound circuits, similar to Figs. 99, 100, 101, and 102.

CHAPTER VII

VARIABLE CIRCUIT CONSTANTS

In the preceding chapters the fundamental laws of transient electric phenomena are derived under the assumption that in any given circuit the resistance, inductance, conductance and condensance, the so-called *circuit constants*, remain constant in value during the transition period under discussion. The transients are due to changes in circuit condition or in the impressed voltage, but during the period required for the dissipation of the stored energy, or the readjustment of the energy content in the system the values of R, L, G and C are assumed constant. The oscillograms, Chaps. III to VI inclusive, of electric transients were obtained from circuits in which the resistance, inductance, conductance and condensance remained essentially constant.

It is evident that if the circuit constants do not remain constant during the period the transients occur but vary rapidly over a wide range of values the nature of the resulting electric phenomena must be correspondingly more complex. The laws for the variations in R, L, G and C are not always known or are so complex that they can not be represented in the form of equations. For example, data for the quantitative ratios between the magnetomotive force and the resulting magnetic flux in iron clad circuits, as indicated by the hysterises loop, may readily be obtained experimentally but it has not been possible to express the relation in the form of a mathematical equation. The empirical equations in common use are limited in their application and give only approximate values.

Variable Resistance.—Change in temperature is the most important factor in producing variations in the resistance of electrical conductors, *the R circuit constant.* For

metals the specific resistance is a linear function of the temperature over a fairly wide range.

$$\rho_t = \rho_0 + at^\circ \tag{270}$$

ρ_t = specific resistance at t° C.
ρ_0 = specific resistance at 0° C.
a = temperature coefficient.

For rapid changes in temperature the rate of change in the resistance may be large. This is illustrated by the oscillograms in Figs. 103, 104. For the tungsten incandescent lamp, Fig. 103, a starting transient appears in the current due to a rapid increase in the resistance of the fila-

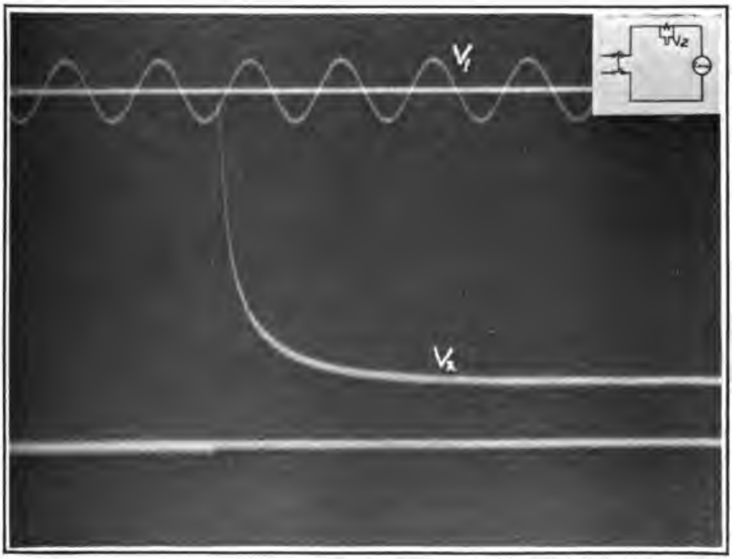

FIG. 103.—Starting current transient of a 60-watt, 120 volts, tungsten incandescent lamp. Resistance variable; timing wave 100 cycles.

ment as the temperature rises. When the switch is closed the filament is at room temperature and the resistance low. The current flowing through the lamp rapidly heats the filament to incandescence with an accompanying increase in the resistance and a decrease in the current. The timing

wave shows that it required about 0.02 of a second for the lamp to reach full brilliancy. During this period the resistance of the filament increased by 400 per cent of its initial value.

For carbon the resistance decreases with an increase in temperature, or the temperature coefficient is negative, as illustrated in Fig. 104, showing that the time required for the resistance to reach a constant value was approximately 0.5 of a second and that the resistance of the incandescent carbon filament is about 70 per cent of its value at room temperature.

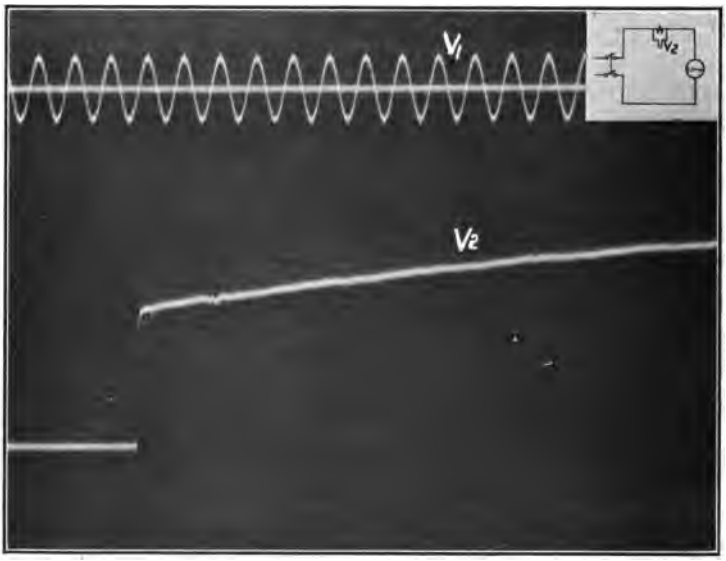

FIG. 104.—Starting current transient of a 50 watt, 120 volts, carbon incandescent lamp. Resistance variable; timing wave 100 cycles.

The temperature of the lamp filament increases until the dissipation of heat by radiation from the lamp is equal to the heat generated by Ri^2 losses. For a direct current supply with constant impressed voltage the constant temperature condition is quickly reached. For alternating currents the power supplied to the lamps pulsates with double

the current frequency and as the lamp emits or radiates heat continuously the temperature, and therefore the resistance of the filament, pulsates. This is illustrated by the oscillogram in Fig. 105. Alternating currents are impressed on two pairs of tungsten and carbon lamps, arranged as shown in the circuit diagram, with the vibrator of the oscillograph in the bridge connection. Since the resistance of

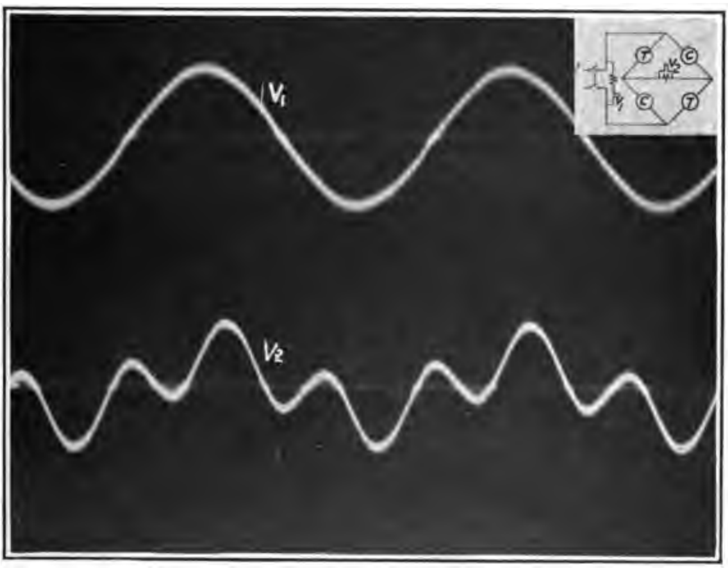

FIG. 105.—Pulsating resistance of tungsten and carbon lamps, alternating currents; 60 cycle supply.

the tungsten lamp increases and the carbon lamp decreases with an increase in temperature, the pulsations in the Ri^2 losses unbalance the bridge as indicated by the pulsations in the currents flowing through the vibrator.

The resistance of the electric arc depends on many factors and may vary over a wide range with extreme rapidity. Since the resistance of the arc decreases with the increase in temperature the arc alone is unstable and hence must be provided with a "ballast" to make continuous operation possible. On alternating currents an inductance placed in

series with the arc serves as the stabilizer and the variations in the resistance of the arc are counterbalanced by the induced voltage in the inductance. In direct current arc lamps a series resistance serves the same purpose.

In commercial systems the electric arcs that affect the series resistance, *the R circuit constant,* occur chiefly in the opening of switches. In breaking the circuit under load, especially when a large quantity of energy is stored magnetically in the circuit, arcs form in which the resistance varies rapidly from zero at start to infinity when the circuit is open.

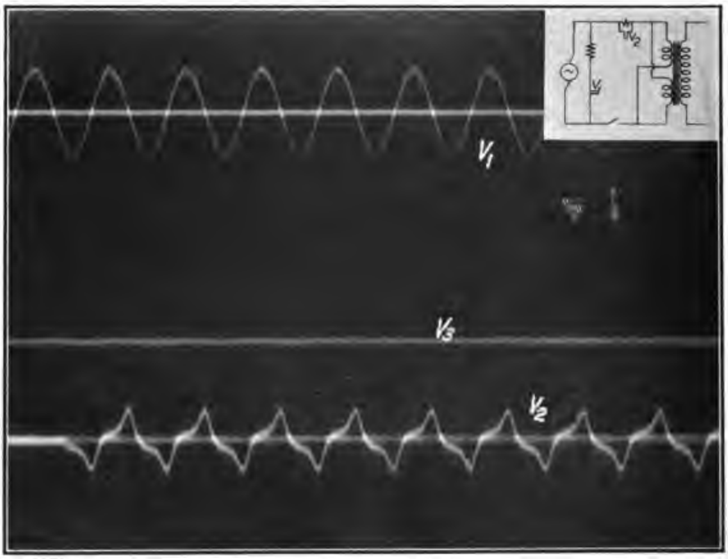

Fig. 106.—Transformer magnetizing current; no starting transient. v_1 = 106 volts; v_2 = primary current; v_3, calibration current = 10.0 amps.; 10 kva. transformers; f = 60 cycles.

This is illustrated by the oscillogram in Fig. 110. In the opening of the switch an arc forms whose resistance rapidly increases, approaching infinity when the circuit opens, which occurs at the point of maximum value in the voltage curve. The increase in the resistance can be determined quantitatively from the oscillogram by combining

data from the rapidly increasing voltage and decreasing current curves.

Variable Inductance.—In iron-clad circuits as in transformers the magnetic flux is not directly proportional to the ampere turns or magnetizing force. Hence the inductance, *the L circuit constant* is not constant but varies with the permeability of the iron. Moreover, the variation in the inductance is different for decreasing and increasing flux

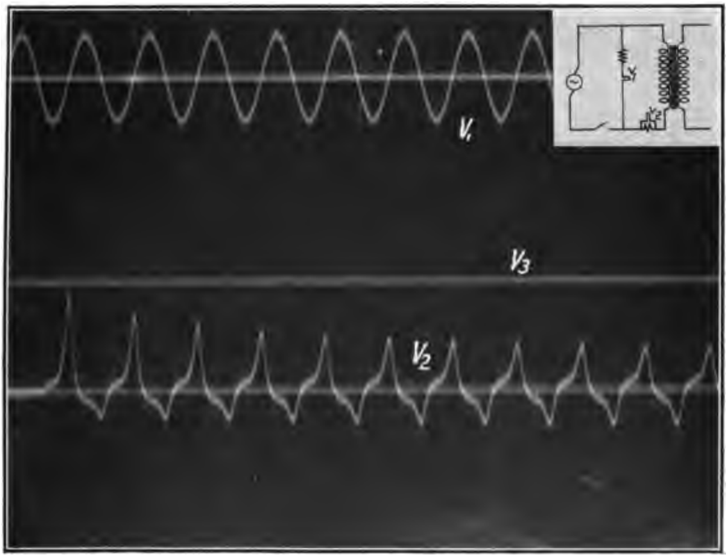

FIG. 107.—Starting transient of magnetizing current in iron-clad circuit. v_1, primary voltage = 236 volts; v_2 = primary current; v_3, calibration current = 13.0 amps.; 10 KVA. transformers; f = 60 cycles.

values and depends on the maximum flux density as indicated by the form of the hysteresis loop. As no satisfactory mathematical expression has yet been found for the hysteresis cycle, solutions of practical problems are obtained by a series of approximations. As a first step in obtaining the shape of transients in iron-clad circuits, neglecting the difference between increasing and decreasing flux values, Fröhlich's formula is generally used.

$$\frac{H}{B} = v + \sigma H \tag{271}$$

The formula is based on the assumption that the permeability of the iron is proportional to its remaining magnetizability and states that the reluctivity of an iron-clad circuit is a linear function of the field intensity.

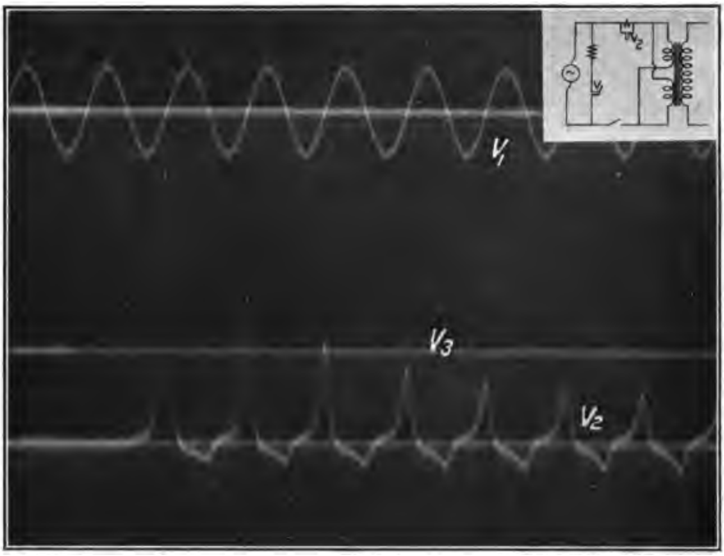

FIG. 108.—Starting transient of magnetizing current in iron-clad circuit. v_1, primary voltage = 106 volts; v_2, primary current; v_3, calibration current = 10.0 amps.; 10 KVA. transformer; f = 60 cycles.

The effect of variable inductance in iron-clad circuits may be illustrated by the starting transients of alternating current transformers. The magnitude of the starting current transient depends more on conditions affecting the value of the inductance in the circuit than on what point on the voltage cycle the switch is closed. The direction and magnitude of the residual magnetism are important factors as a combination of much residual flux with an additional magnetizing force in the same direction may bring the flux density in the core beyond the saturation point and hence greatly reduce the inductance in the circuit.

For the oscillograms in Figs. 106 to 109 a constant alternating current voltage of sine wave shape was impressed on

the transformer terminals. In Figs. 106, 107 and 108 the residual magnetism in the iron core was, in each case, removed before the oscillogram was taken. The three oscillograms form a series showing the transient current due to the closing of the switch at different points of the voltage cycle. In Fig. 106 the switch was closed at an instant the magnetizing current would have been zero (maximum point on the voltage wave), if the circuit had been closed earlier,

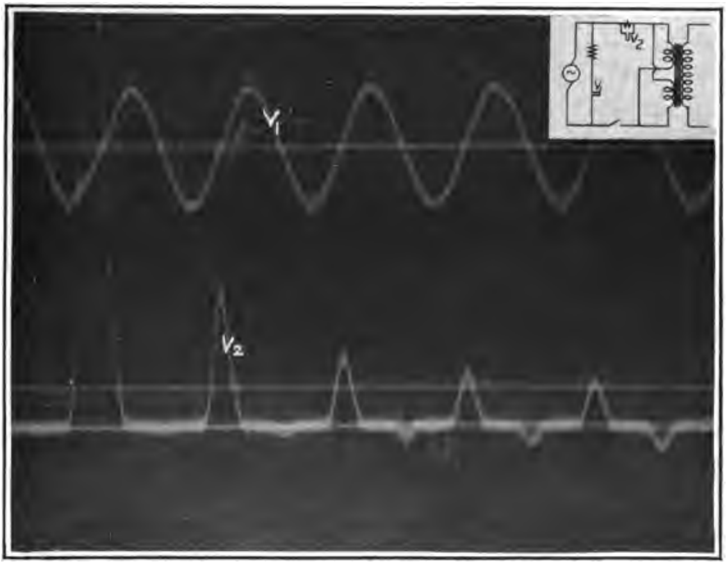

FIG. 109.—Starting transient in transformer magnetizing current. Residual magnetism.
v_1, primary voltage = 150 volts; v_2, primary current; v_3, calibration current = 2.5 amps.; f = 60 cycles.

and hence no starting transient. In Figs. 107 and 108 the switch was thrown at other than the zero point of the magnetizing current cycle. The impressed voltage was less than normal and the change in the flux density is not large and hence the inductance at the maximum points of the magnetizing current wave is practically constant. The starting current transients under the given conditions may be expressed by an exponential equation as explained in Chap. IV.

The starting transient in Fig. 109 differs greatly both in form and magnitude, as compared to Fig. 108, although the circuits were closed in the two cases at approximately the same point on the voltage wave. In Fig. 109 the impressed voltage was higher than the rating of the transformer and the residual magnetism in the iron core was in the same direction as the flux produced by the magnetizing current during the first half cycle. Above saturation of the iron

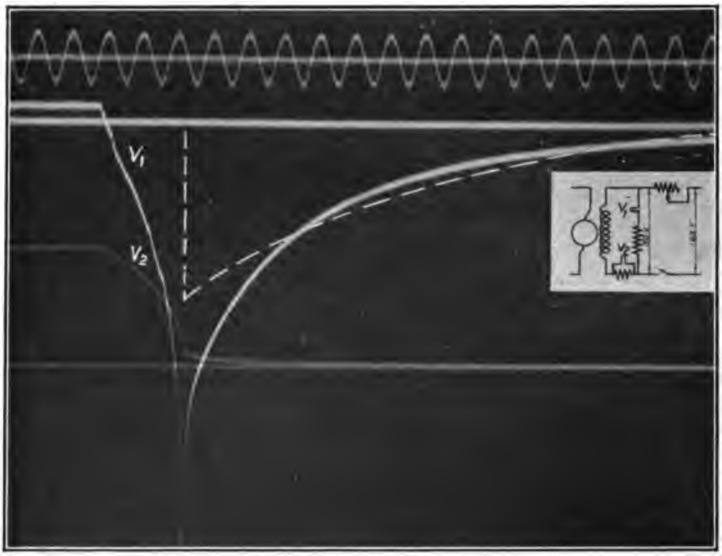

FIG. 110.—Breaking generator field circuit. Field current and voltage transients.
v_1 = 100 cycle timing wave; v_2, impressed voltage = 31.5 volts; v_3, field current = 4.0 amps.

core the transformer inductance is relatively small and hence the first half cycle shows a correspondingly large current transient. A smooth curve drawn through the successive maximum values of the starting transient in Figs. 107 or 108 could with a fair degree of accuracy be expressed by the exponential equation; but the corresponding curve drawn through the successive maximum values of the current wave in Fig. 109 would have a much steeper

gradient due to the variation in the inductance, L, of the transformer winding.

The same effect, due to variable inductance, may be obtained in breaking the field circuit of a direct current generator as illustrated by the oscillogram in Fig. 110. The change in the voltage and current curves from the instant the jaws of the switch separate to the peak value of the voltage is largely due to a change in the arc resistance. After the arc breaks, at the peak of the voltage curve, the

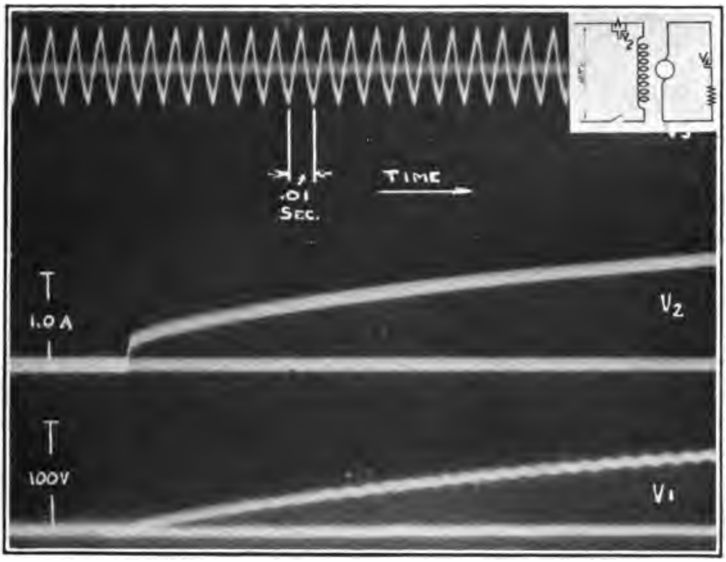

FIG. 111.—Building up generator field. Field current and armature voltage transients.
v_1 = generator terminal voltage; v_2 = field current; v_3 = 100 cycle timing wave.

vibrator circuit provides a path for the dissipation of the energy stored in the field. As the resistance in the vibrator circuit is constant the voltage curve also represents the transient current. The dotted curve traced on the oscillogram shows the exponential curve conforming with the latter part of the actual voltage or current curves. The relative magnitude of the peak value of the voltage to the

corresponding initial value of the dotted curve indicates the change in magnitude of the inductance in the field winding.

The corresponding variation in the inductance when the generator field is formed is evidenced by the starting field current and armature voltage curves shown in Fig. 111.

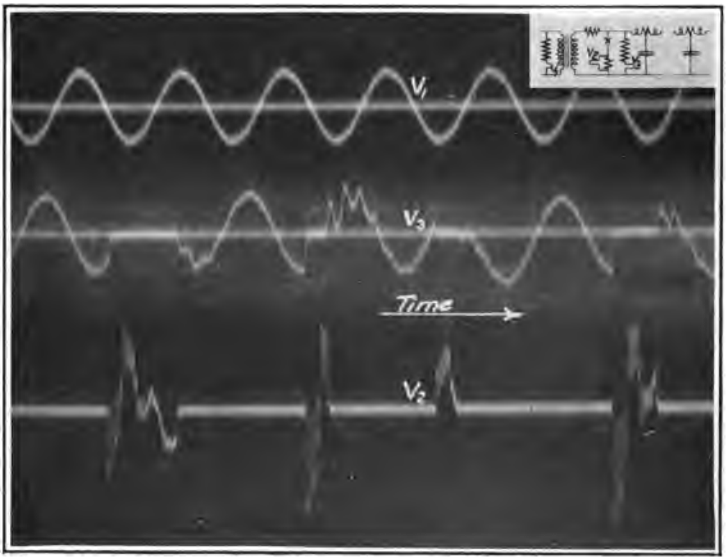

FIG. 112.—Arcing grounds on transmission line.
Ground at generator end. Impressed voltage = 90 volts; f = 60 cycles; v_2 = arc voltage; v_3 = arc current.

Variable Conductance.—In the calculations on power transmission lines and in general for constant potential systems in good condition the leakage through the insulation is small, so that the conductance is negligible and *the G circuit constant* may be taken as equal to zero. The insulation of electric circuits deteriorate with varying rates and the conductance and leakage increase and may become very large, as for example, if the insulation completely breaks down and a short circuit is formed. A rupture of the insulation or any sudden change in the conductance of the electric circuit will of necessity cause violent disturbances in the

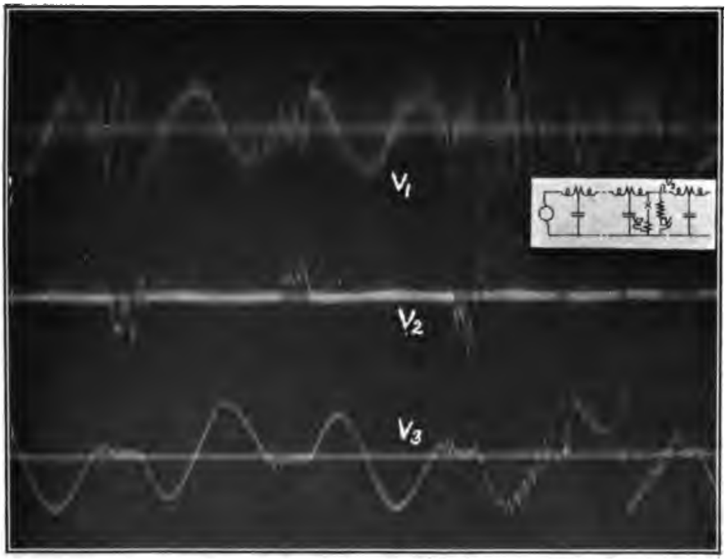

Fig. 113.—Arcing grounds on transmission line.
Semi-continuous copper-carbon arc 114 miles from generator end of 207 mile artificial transmission line. 4/0 copper, 96 in. spacing. v_1 = arc voltage; v_2 = arc current; v_3 = line current.

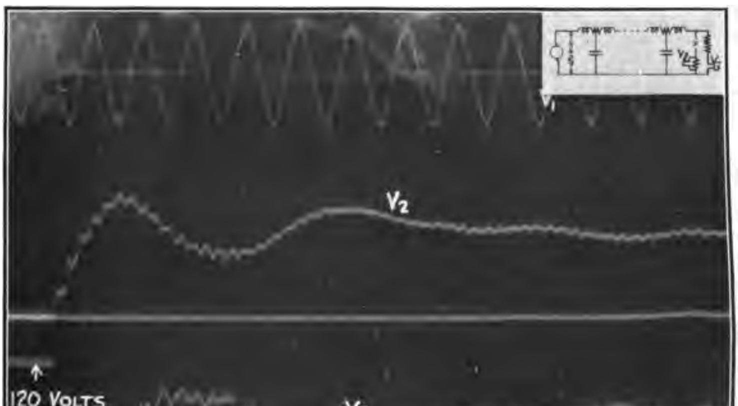

Fig. 114.—Arcing grounds on transmission line.
Arc at receiver end. v_1 = 100 cycles timing wave; v_2 = current receiver end; v_3 = voltage receiver end.

system. Arcing grounds or intermittent arcs, as illustrated by the oscillograms in Figs. 112 to 115, are prolific sources of electric transients. It is evident that momentary short circuits, as would be produced by an intermittent arcing ground with the conductance varying practically from zero to infinity at an extremely rapid rate, would give rise to oscillations of any frequency and produce waves and impulses that would travel to all parts of the system.

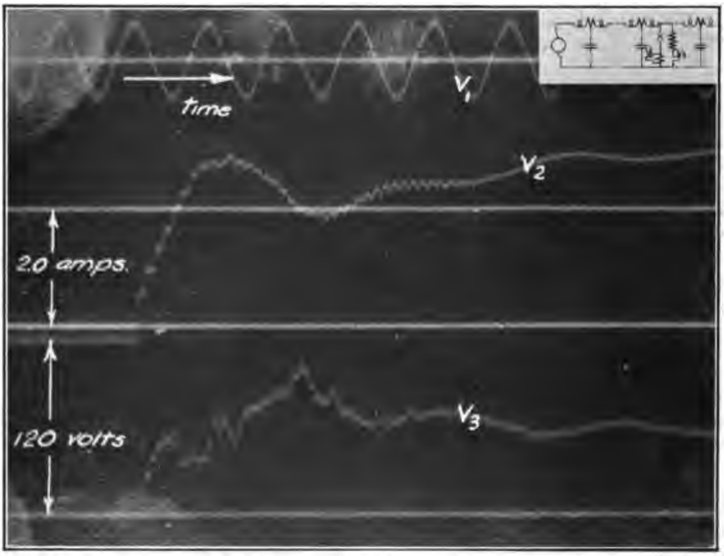

Fig. 115.—Arcing grounds on transmission line.
Arc at middle of line. v_1 = 100 cycle timing wave; v_2 = arc current; v_3 = arc voltage.

Variable Condensance.—Under ordinary conditions and for low voltages, air is very nearly a perfect insulator. In other words, the conductivity of air is practically zero, the permittivity, unity and the energy loss extremely small. If the voltage is increased until the limit of the insulating strength of the air is reached important changes occur in both the electric and dielectric circuit constants. With the occurrence of visual corona in high voltage circuits the conductivity of the air in the space filled by the corona is

increased. Thus in circuits with parallel wires as high tension transmission lines a voltage gradient above 29.8 ky. per cm. will produce corona in the air surrounding the conductor surface and this space filled by the corona glow becomes semi-conducting. This produces a change in the circuit condensance as with the appearance of the corona the effective size of the conductor, and hence of the condenser surface, is increased. For alternating currents the visual

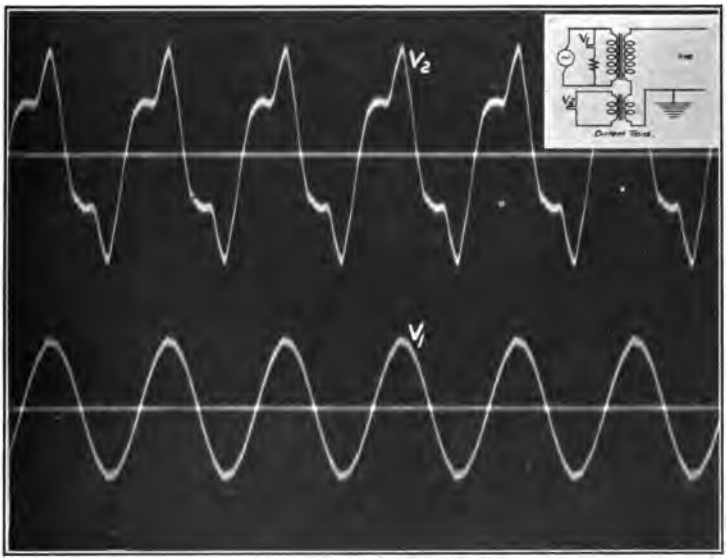

FIG. 116.—Variable condensance. Corona.
Single phase line 135 ft. long, 10 in. spacing, No. 24 A.W.G. steel wire. Line voltage = 3400 volts; line current = 0.0008 amps.

corona is intermittent, appearing only near the peaks of the successive voltage waves, when the instantaneous voltage gradient exceeds 29.8 kv. per cm., the required value for producing visual corona. As a consequence the condensance of the alternating current circuit when corona occurs is variable, pulsating with double the frequency of the voltage. This is illustrated by the oscillograms in Figs. 116, 117. If an alternating current voltage of sine wave shape is impressed on a circuit having constant con-

densance the charging current would also follow the sine law. If the condensance, *the C circuit constant*, varies during the voltage cycle, a corresponding change is produced in the wave shape of the charging current.

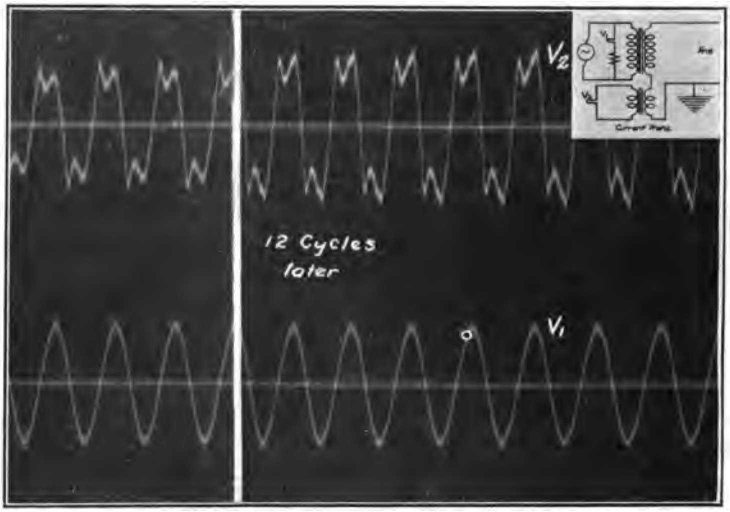

FIG. 117.—Variable condensance. Corona.
Line covered with snow and swaying in the wind. Line constants same as for Fig. 116.

Problems and Experiments

1. Take oscillograms, similar to Figs. 106, 108 and 109, showing the starting transients of transformers.

2. Take oscillograms showing the variable condensance of an arcing ground for direct and alternating currents on a transmission line.

3. Take oscillograms similar to Figs. 116 and 117, showing the change in condensance produced by corona.

4. Take an oscillogram similar to Fig. 110, showing the voltage across the terminals. Compare the operating voltage with the maximum value when the switch is opened.

CHAPTER VIII

RESONANCE

Electric resonance phenomena have essentially permanent or stable characteristics but are closely related to, and frequently accompanied by, true electric transients. The conditions required for producing resonance and expressions for the frequency at which resonance occurs, in simple electric circuits, are referred to in Chap. IV in connection with the derivation of the equations for the natural frequency of free oscillations. Resonance in an electric circuit implies a forced oscillation of energy between the magnetic and dielectric fields, during which the energy dissipated as heat by the Ri^2 and Ge^2 losses, is supplied from some outside source. Distinction is usually made between *voltage resonance* occurring in series circuits, and *current resonance* that may be produced in parallel circuits.

Voltage Resonance.—In series circuits *voltage resonance occurs at that frequency of the impressed voltage for which the impedance of the circuit is a minimum.* In series circuits, as in Fig. 118, the impedance is a minimum when the condensive and inductive reactances are equal.

$$ {}_Lx = {}_cx;\ 2\pi fL = \frac{1}{2\pi fC} \tag{275}$$

$$ f = \frac{1}{2\pi\sqrt{LC}} \tag{276}$$

$$ z = \sqrt{R^2 + ({}_Lx - {}_cx)^2} = R \tag{277}$$

Frequently the assumption is made that a circuit is in resonance when the current and the impressed voltage are in phase, as illustrated by the vector diagram in Fig. 119. For straight series circuits the conditions required for unity power factor of the power supplied to the circuit are iden-

tical with the requirements for minimum impedance, but in complex circuits or for current resonance in parallel circuits this is not always the case.

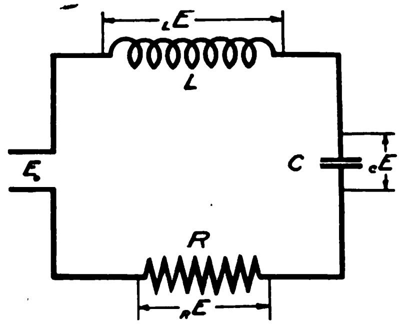

FIG. 118.—Series circuit for voltage resonance.

Equation (276) gives the optimum condition for resonance in series circuits for given values of the R, L and C, the circuit constants. Resonance phenomena are, however,

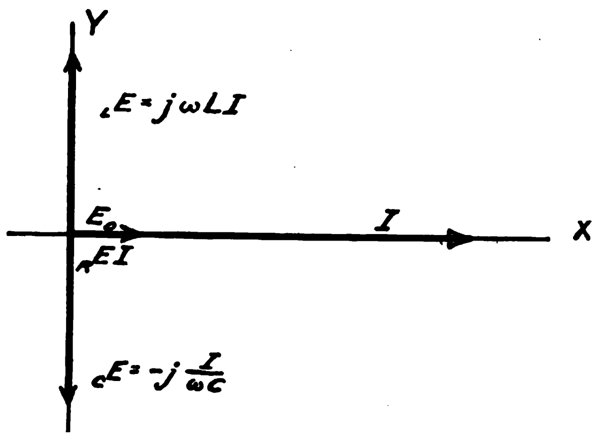

FIG. 119.—Vector diagram for voltage resonance in series circuit Fig. 118.

not limited to the exact frequency determined by equation (276), but persist over a range of frequencies, more or less sharply defined, depending on the relative magnitude of the resistance and the inductive or condensive reactance. The voltage-frequency relation for given constant values of R, L and C, is shown in Fig. 120. The feature of special

interest is the large increase in ${}_LE$ and ${}_cE$, the voltages across the inductance and the condensance under resonance conditions. If the resistance is small ${}_LE$ and ${}_cE$ may rise

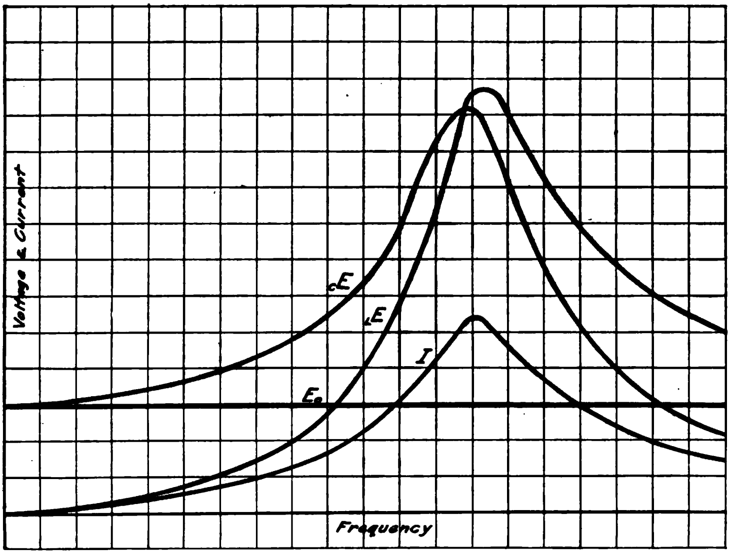

FIG. 120.—Voltage resonance for series circuit as in Fig. 118.

to many times the value of the impressed voltage E_0. Voltage resonance in power circuits is undesirable as the increase in voltage above the normal operating value endangers the insulation.

The effect of varying the resistance on the *sharpness of resonance* is illustrated by Fig. 121. The smaller the resistance the higher and sharper the voltage and current resonance peaks. The sharpness of resonance may be defined as the ratio of the inductive reactance or the condensive reactance at resonance frequency to the resistance in the circuit.

$$\text{Sharpness of resonance} = \frac{{}_Lx}{R} = \frac{{}_cx}{R} \qquad (278)$$

Reactance Curves.—Curves in rectangular coordinates showing graphically the changes in magnitude of the

inductive reactance and the condensive reactance produced by varying the frequency of the impressed voltage are of much value for giving a clear insight into resonance phenomena. The ordinates of the curves in Fig. 122 represent respectively the inductive reactance, ${}_{L}x$, the condensive reactance, ${}_{c}x$, and the total reactance, x, with the frequency of impressed voltage as the other variable represented

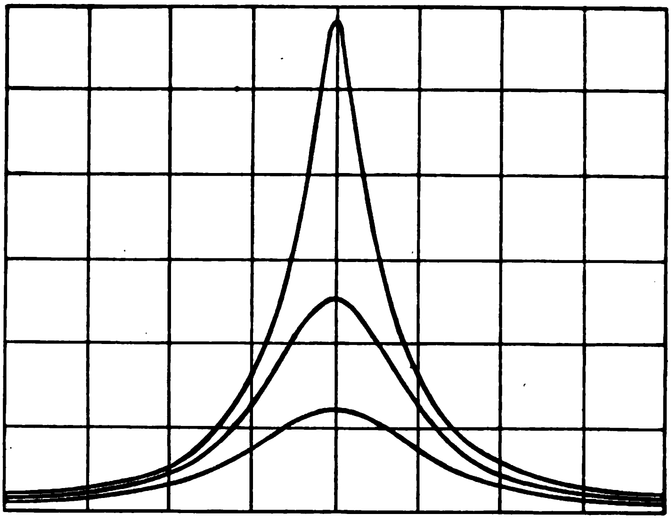

FIG. 121.—Resonance curves for series circuit with different resistances.

along the X axis. Since resonance occurs when the impedance of the series circuit is a minimum, the resonance frequency is indicated by the intersection of the total reactance curve, in Fig. 122, with the X axis.

Current Resonance.—Forced oscillatory transfer of energy between dielectric and magnetic fields is the basis of resonance phenomena in parallel circuits in much the same manner as in series circuits, but the resultant voltage and current values are different. In simple parallel circuits, as illustrated by Figs. 123 and 127, *current resonance occurs at that frequency of the impressed voltage for which the total admittance is a minimum.* In discussions of resonance phenomena it is frequently assumed that the

conditions for current resonance in parallel circuits are met when the inductive and condensive susceptances are equal, that is, when the impressed current and voltage are in phase. That this assumption is not in full accord with the above definition of current resonance for all

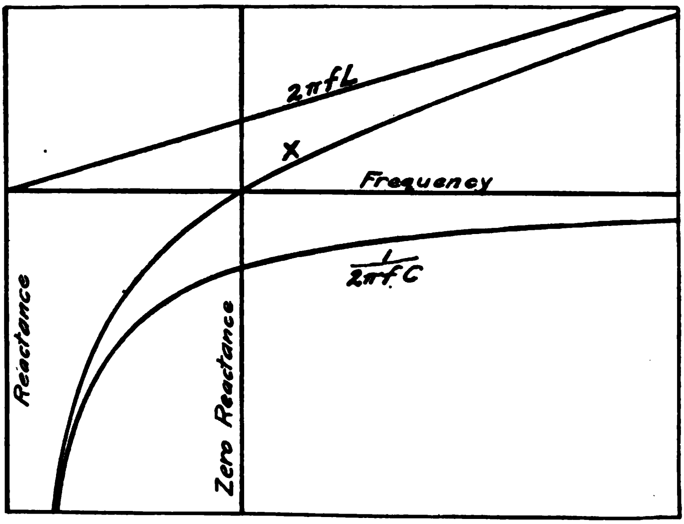

FIG. 122.—Reactance curves. Series circuit.

values of R in the circuits shown in Figs. 123 and 127, may readily be seen from the corresponding vector diagrams in Figs. 124 and 128. For the circuit in Fig. 123 current resonance occurs when ${}_Lb = {}_cb$ under the condition that $R = O$. From the vector diagram in Fig. 124:

$$ {}_L\dot{I} = E_0(g - j_Lb) = E_0 \frac{R - j\omega L}{R^2 + \omega^2 L^2} \tag{279} $$

$$ {}_c\dot{I} = j_cb\dot{E}_0 = j\omega C\dot{E}_0 \tag{280} $$

$$ \dot{I} = {}_L\dot{I} + {}_c\dot{I} = \dot{E}_0[g + j({}_cb - {}_Lb)] \tag{281} $$

$$ I = E_0\sqrt{g^2 + ({}_cb - {}_Lb)^2} \tag{282} $$

$$ = E\sqrt{\left(\frac{R}{R^2 + \omega^2L^2}\right)^2 + \left(\omega C - \frac{\omega L}{R^2 + \omega^2L^2}\right)^2} \tag{283} $$

The total current, I, will be in phase with the impressed voltage, E, if

$$ {}_cb = {}_Lb; \omega C = \frac{\omega L}{R^2 + \omega^2 L^2} \tag{284} $$

Hence for unity power factor supply, the frequency for the circuit in Fig. 123,

$$ f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \tag{285} $$

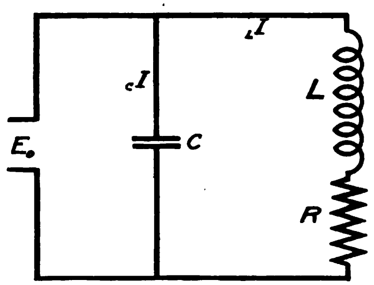

FIG. 123.—Parallel circuit for current resonance.

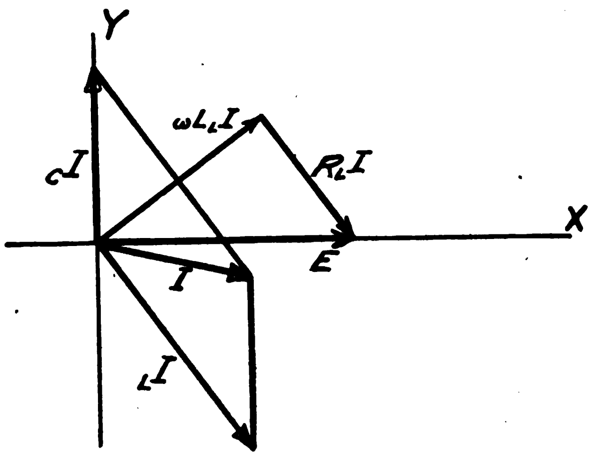

FIG. 124.—Vector diagram for circuit in Fig. 123.

For maximum current resonance the total admittance of the circuit must be a minimum and hence for constant impressed voltage, E_0, the total current must be a minimum. Therefore, the resonance frequency may be obtained by

equating the first derivative of I to ω, L, or C, as the case may be, in equation (283) to zero. Taking ω as the variable factor with R, L, C, and E constants for the circuit in Fig. 123:

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{C^2L^2} + \frac{2R^2}{CL^3}\right)^{1/2} - \frac{R^2}{L^2}} \tag{286}$$

Letting C be the variable factor with R, L, ω, and E constant:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \tag{287}$$

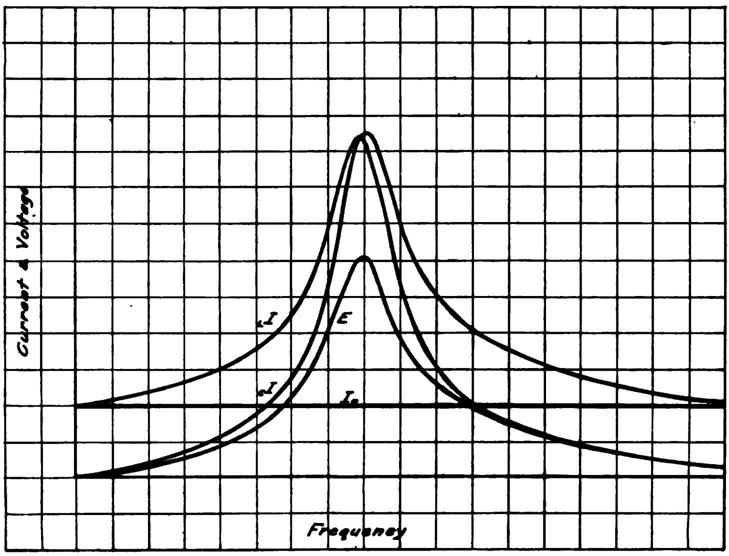

FIG. 125.—Current resonance. Variable ω. For Fig. 123, Equation (286).

Letting L be variable with R, C, ω, and E constants:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{2LC} + \left(\frac{R^4}{L^4} + \frac{R^2}{CL^3} + \frac{1}{4L^2C^2}\right)^{1/2}} \tag{288}$$

In a similar manner expressions may be obtained for *unity power factor frequency* and *maximum current resonance frequency* for ω, C or L respectively as the variable with the other factor constants for the circuit in Fig. 127.

$$ {}_L\dot{I} = \dot{E}_0(g - j_Lb) \tag{289}$$
$$ {}_c\dot{I} = \dot{E}_0(G + j_cb) \tag{290}$$
$$ \dot{I} = {}_L\dot{I} + {}_c\dot{I} = \dot{E}_0[(g + G) + j({}_cb - {}_Lb)] \tag{291}$$
$$ I = E_0\sqrt{\left(\frac{R}{R^2 + \omega^2L^2} + G\right)^2 + \left(\omega C - \frac{\omega L}{R^2 + \omega^2L^2}\right)^2} \tag{292}$$

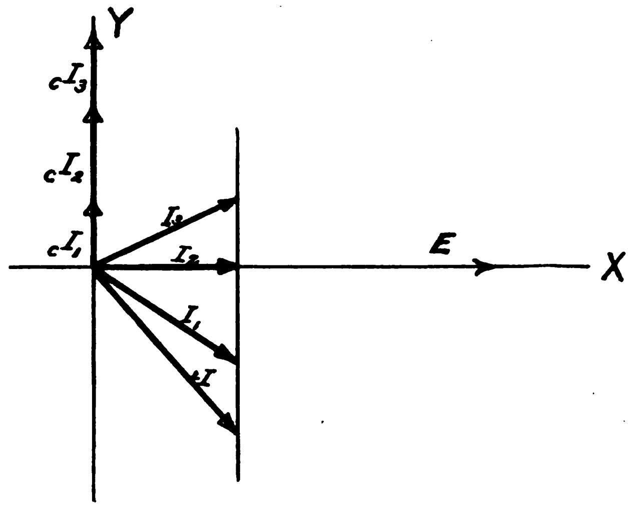

FIG. 126.—Vector diagram. Variable C. For Fig. 123, equation (287).

The total current, I, will be in phase with the impressed voltage, E_0 if

$$ {}_cb = {}_Lb;\ \omega C = \frac{\omega L}{R^2 + \omega^2L^2} \tag{293}$$

Hence, the frequency required to give unity powerfactor for the circuit in Fig. 127 is the same as for Fig. 123.

$$ f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \tag{294}$$

The frequency for maximum current resonance if ω is variable while R, L, C, G and E_0 are constant, Figs. 127, 128:

$$ f = \frac{1}{2\pi}\sqrt{\left(\frac{1 - 2RG}{C^2L^2} + \frac{2R^2}{CL^3}\right)^{\frac{1}{2}} - \frac{R^2}{L^2}} \tag{295}$$

If C be the variable, while R, L, G, ω and E_0 are constant:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \tag{296}$$

If L be the variable, while R, C, G, ω and E_0 are constant:

$$= \frac{1}{2\pi}\sqrt{\frac{1+2RG}{2LC} + \left[\frac{R^4}{L^4} + \frac{R^2+2RG}{CL^3} + \frac{(1+2RG)^2)}{4L^2C^2}\right]^{1/2}} \tag{297}$$

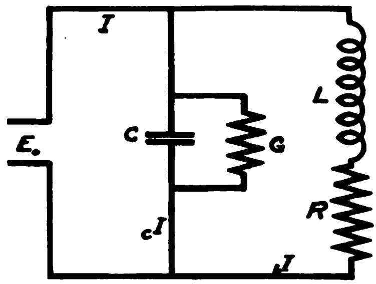

FIG. 127.—Parallel circuit with leaky condenser.

In tuning ratio receiver sets resonance is obtained by varying C or L as expressed by equations (296) (297).

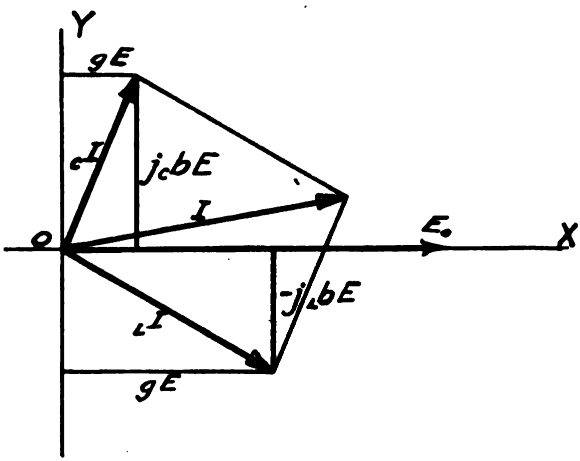

FIG. 128.—Vector diagram for circuit in Fig. 127.

Changes in the inductance by varying the number of turns, also changes the ohmic resistance but the conditions

required for equation (297) may be obtained experimentally for circuits in which the change in L may be produced by varying the mutual or self-induction between parts of the inductance in circuit.

The smaller the resistance in the resonating circuit the greater the increase in the resonance current and voltage. *Resonance phenomena are of commercial importance only when the resistance in circuit is small as compared to the inductance and condensance.*

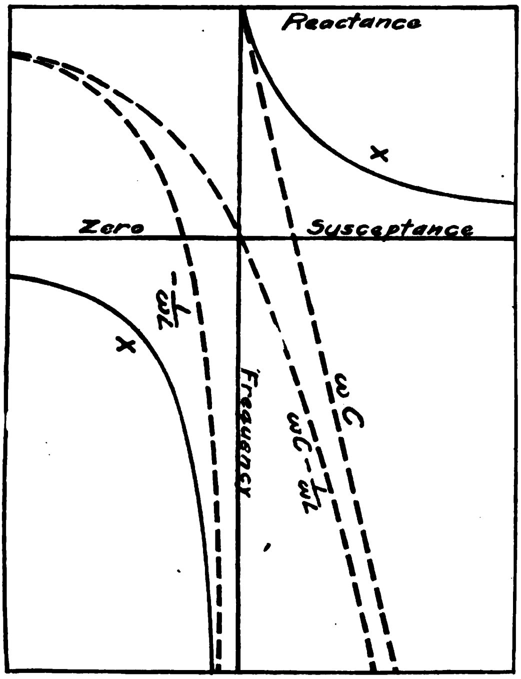

FIG. 129.—Susceptance curves for parallel circuit.

In most cases and particularly those of greatest importance, the resistance is negligibly small. If R and G are taken equal to zero all the resonance frequency equations (295) to (297) become identical in form.

Resonance frequency, massed circuit constants (approximate value):

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{298}$$

In commercial work equation (298) is in general use, giving with sufficient accuracy the resonance frequency for simple circuits having *massed condensance, inductance* and *resistance.*

For distributed circuit constants, as in long transmission lines, the space distribution of the voltage and current waves must be taken into consideration, the approximate resonance frequency is given by equation (299), as explained in Chap. VI on Transmission Line Oscillations.

Resonance frequency, uniformly distributed circuit constants (approximate value)

$$f = \frac{1}{4\sqrt{LC}} \tag{299}$$

In power circuits resonance conditions must be avoided or the resistance in circuit be sufficiently large to prevent any marked increase due to resonance in the current and voltage.

Coupled Circuits.—Resonance phenomena are of fundamental importance in the operation of radio communication apparatus. The circuits in commercial use are more complex than the forms discussed above but may be considered as combinations of simple circuits. In general the component simple circuits have certain parts in common.

The couplings or connections may be made in a number of ways. For two circuit apparatus the coupling is generally made in one of the following ways:

1. By direct connection across an inductance coil. Direct coupling as in Fig. 130.
2. By magnetic induction. Inductive or magnetic coupling as in Fig. 131.
3. By dielectric induction. Condensive, capacitative or dielectric coupling as in Fig. 132.

The inductive interaction of the voltages and currents in two resonating coupled circuits and the transfer of the

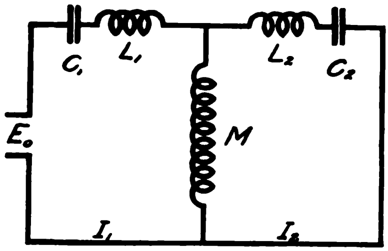

FIG. 130.—Direct coupling.

oscillating energy between the primary and secondary circuits are illustrated by the oscillograms in Figs. 133 to

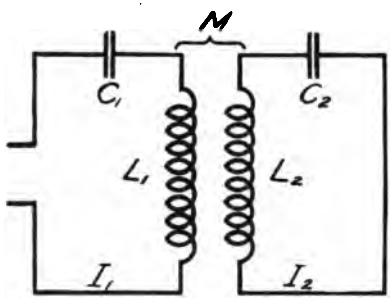

FIG. 131.—Inductive or magnetic coupling.

138. The oscillations of the energy between the dielectric and magnetic fields of each circuit are combined with a

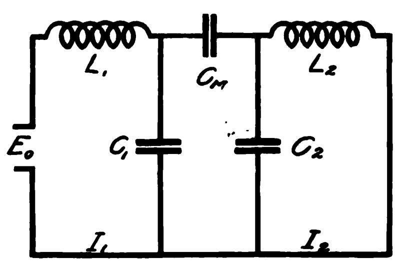

FIG. 132.—Condensive or dielectric coupling.

rapid to and fro transfer of the energy between the magnetically or dielectrically coupled circuits. In Fig. 133 the energy was initially stored in the condenser in the pri-

mary circuit. By closing the switch oscillations are set up between the dielectric and magnetic fields in both the primary and secondary circuits, and these are combined with a rapid to and fro transfer of the energy between the two circuits. The oscillogram shows that the frequency of oscillation between the magnetic and dielectric fields in both the primary and secondary was 790 cycles per second, while the frequency of transfer between the circuits was approximately 99 cycles per second. That is, the time required for the transfer of the energy from the primary to the secondary through the magnetic coupling and back again was approximately equal to eight complete oscillations between the magnetic and dielectric fields of either the primary or the secondary circuits. The oscillations decrease in magnitude due to the Ri^2 losses and practically all of the energy was dissipated into heat in $\frac{1}{20}$ of a second.

For the oscillogram in Fig. 134 the primary circuit was opened at the instant all the energy had been transferred from the primary to the secondary circuit, thus preventing its return to the primary circuit. Hence the secondary continues to oscillate until all the energy has been dissipated as heat by the Ri^2 losses.

The oscillogram in Fig. 135 shows the starting oscillatory transient of two inductively coupled circuits when an alternating current of resonance frequency is impressed on the primary. Similar oscillograms showing the oscillatory transfer of energy between the primary and secondary of dielectrically coupled circuits are shown in Figs. 136, 137 and 138. The difference in form in the three oscillograms is due to change in the degree of coupling as indicated by the quantitative data in each case.

Coupling Coefficient.—In coupled circuits as in Figs. 130 and 131, the interaction will depend on what part of the total magnetic flux interlinks both circuits. The degree of coupling which is often termed "loose" or "close," depending on whether a small or large fraction of the flux interlinks both circuits, is quantitatively expressed by

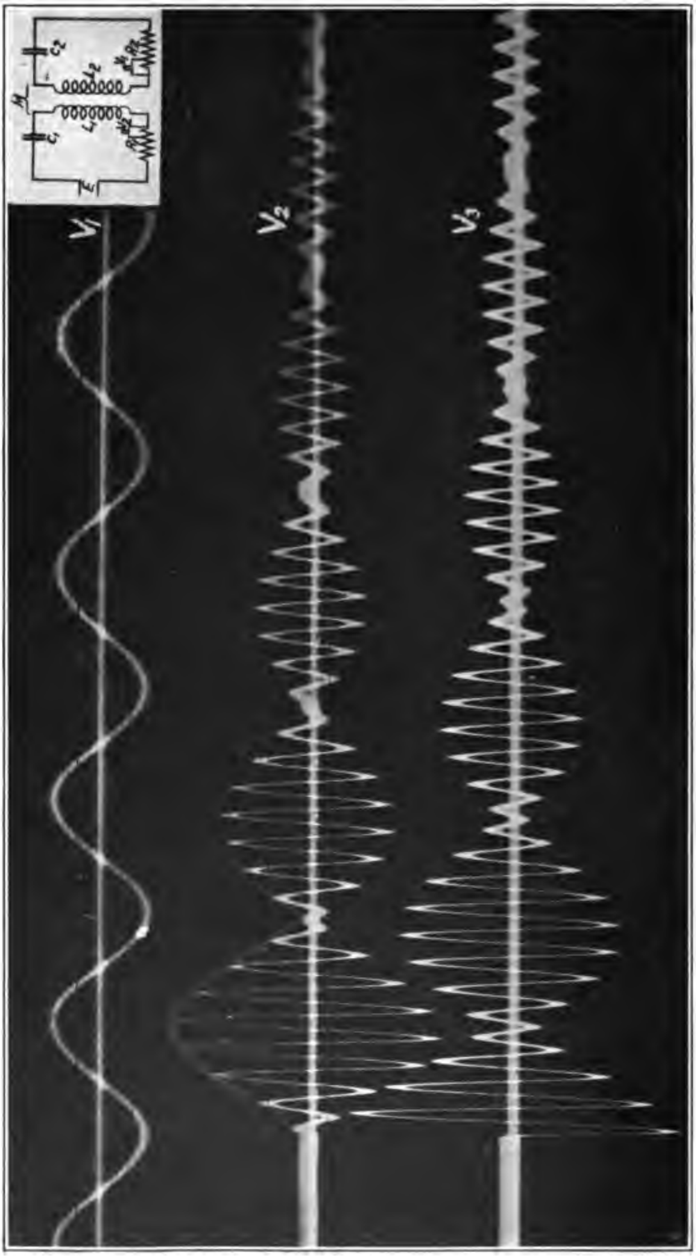

FIG. 133.—Transient oscillations. Inductive coupling. E = 700 volts; R = 5 ohms; L = 0.205 henrys; C = 0.2 microfarads; K = coefficient of coupling = 11 per cent.; timing wave 100 cycles; f = 790 cycles when K = 0.

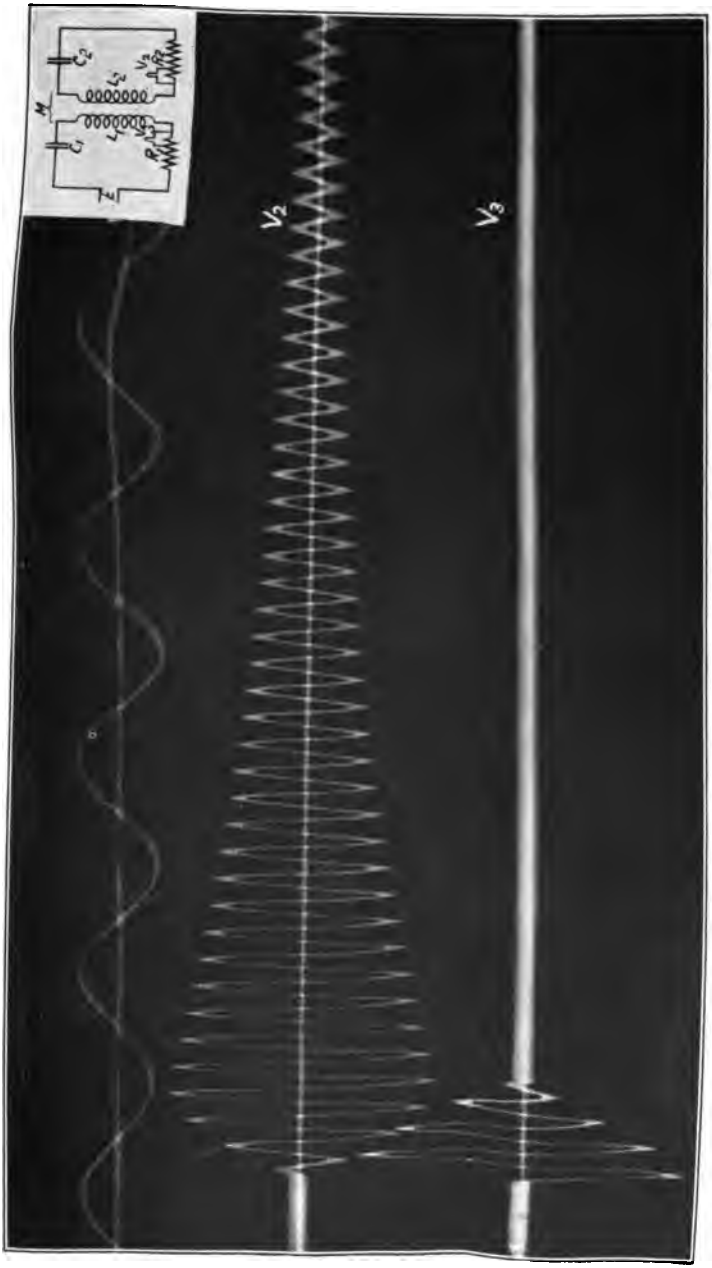

FIG. 134.—Transient oscillations in inductively coupled circuits. Primary opened when the energy has transferred to the secondary circuit. E = 700 volts; R = 5 ohms; L = 0.205 henrys; C = 0.2 microfarads; K = coefficient of coupling = 11 per cent.; timing wave 100 cycles; f = 790 cycles when K = 0.

the coupling coefficient. This is defined as the ratio of the mutual reactance to the square root of the product of the primary and secondary circuit reactances.

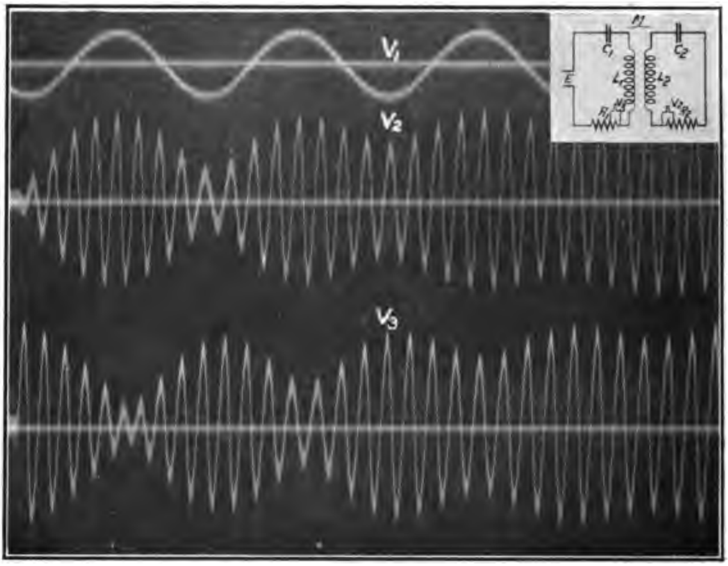

FIG. 135.—Transient oscillations. Inductive or magnetic coupling. Resonant charge.
Impressed frequency = 750 cycles; R = 6.5 ohms; L = 0.205 henrys; C = 0.2 microfarads; coefficient of coupling = 11 per cent; timing wave 100 cycles; natural frequency 790 cycles when K = 0.

Inductive coupling coefficient, Fig. 131:

$$\kappa = \frac{{}_Lx_m}{\sqrt{{}_Lx_1\,{}_Lx_2}} = \frac{M}{\sqrt{L_1L_2}} \tag{300}$$

M = mutual inductance
L_1 = inductance of primary with the secondary open or removed
L_2 = inductance of secondary with the primary open or removed.

Condensive coupling coefficient, Fig. 132:

$$\kappa = \frac{{}_Cx_m}{\sqrt{{}_Cx_1\,{}_Cx_2}} = \frac{\sqrt{C_1C_2}}{C_m} = \sqrt{\frac{C_aC_b}{(C_a + C_m)\,(C_b + C_m)}} \tag{301}$$

C_m = condensance in common condenser

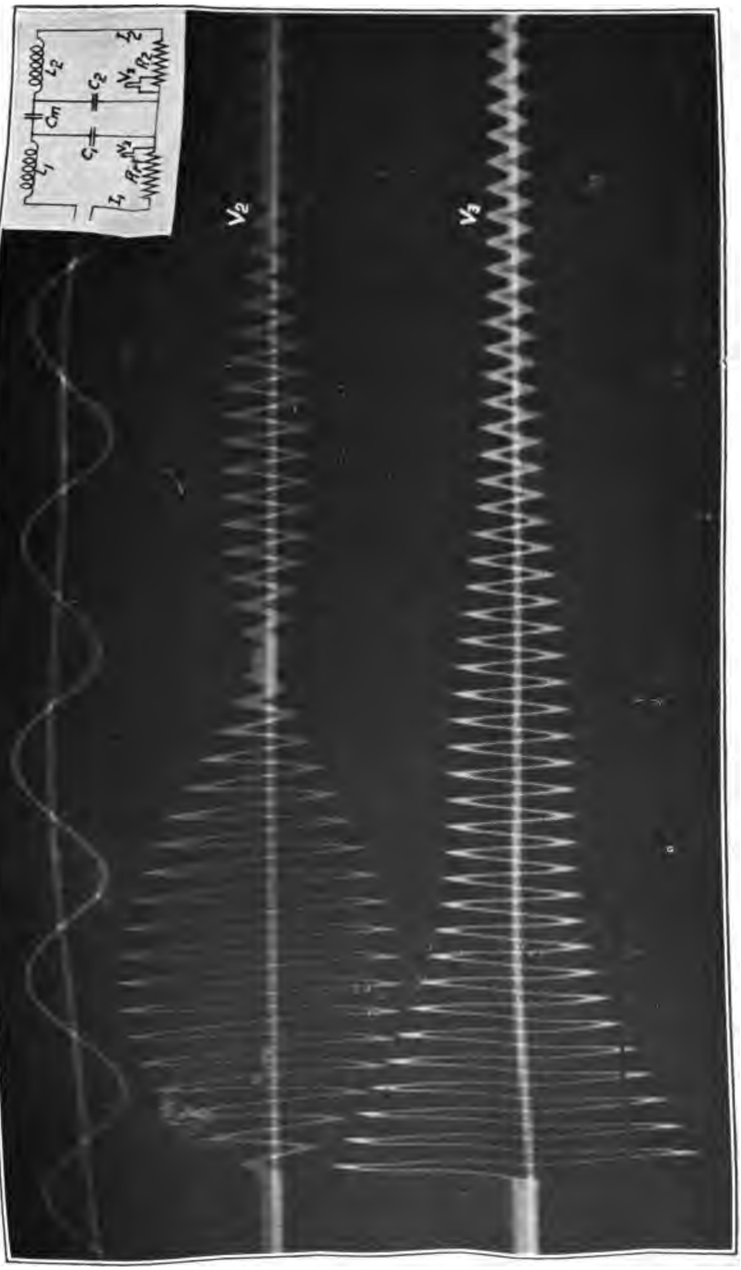

FIG. 136.—Transient oscillations. Condensive or dielectric coupling. E = 700 volts; R = 5 ohms; L = 0.205 henrys; C = 0.2 microfarads; K = coefficient of coupling = 4.5 per cent; timing wave 100 cycles; f = 790 cycles when K = 0.

FIG. 137.—Transient oscillations. Condensive or dielectric coupling. E = 700 volts; R = 5 ohms; L = 0.205 henrys; C = 0.2 microfarads; K = coefficient of coupling = 16.7 per cent; timing wave 100 cycles; f = 790 cycles when K = 0.

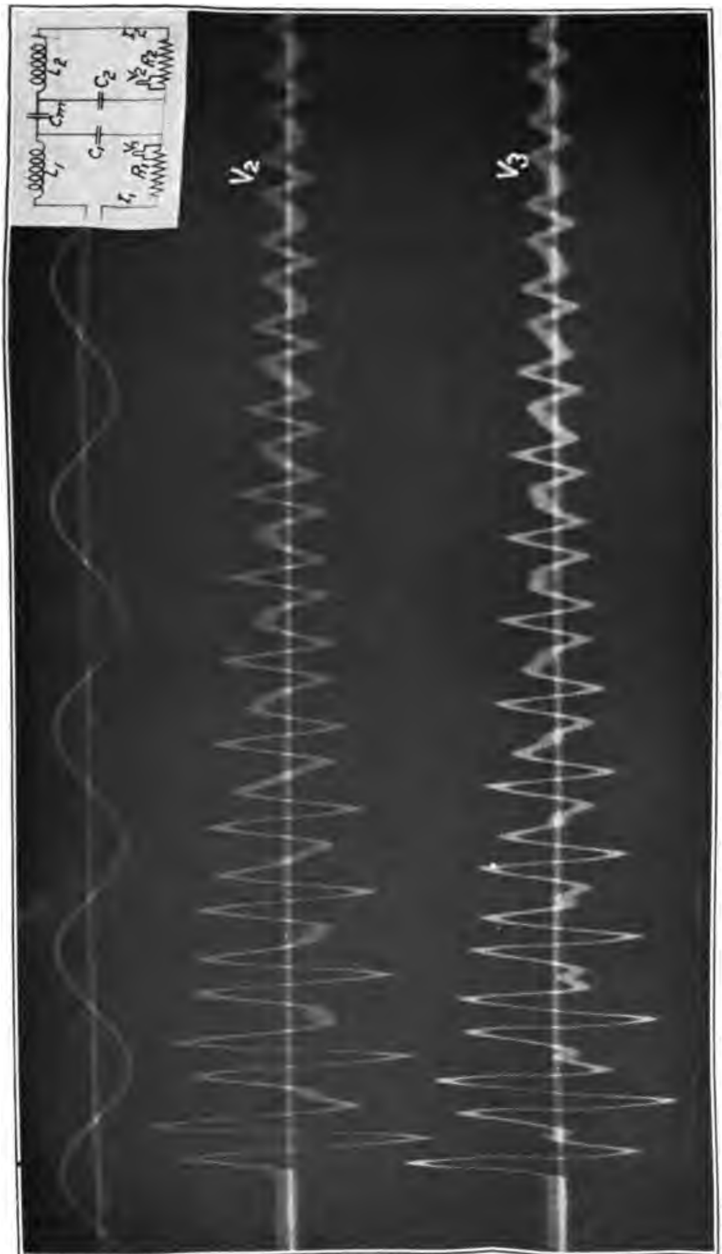

FIG. 138.—Transient oscillations. Condensive or dielectric coupling. E = 700 volts; R = 5 ohms; L = 0.205 henrys; C = 0.2 microfarads; K = coefficient of coupling = 37.5 per cent timing wave 100 cycles; f = 790 cycles when K = 0.

C_a = condensance in primary circuit
C_b = condensance in secondary circuit

$$C_1 = \frac{C_a C_m}{C_a + C_m} = \text{total condensance in primary}$$

$$C_2 = \frac{C_a C_m}{C_b + C_m} = \text{total condensance in secondary.}$$

Multiplex Resonance.—In complex circuits or series of double energy loops the conditions for resonance may be satisfied for more than one frequency of the impressed voltage. The degrees of freedom, or the number of frequencies at which resonance may occur, depends on the number and interconnection of the elemental double energy circuits in the system. Thus, a transmission line having uniformly distributed *R*, *L*, *G* and *C*, and hence to be considered as consisting of an infinite series of infinitesimal double energy circuits, would resonate for the fundamental frequency of the line as a unit and for any multiple or harmonic of the fundamental frequency. As the line constants are not perfectly constant and the distribution of *R*, *L*, *G* and *C* not quite uniform, resonance is limited to the fundamental and a few of the lower harmonics.

Resonance Growth and Decay.—As stated in the beginning of this chapter resonance in electric circuits implies a forced oscillation of energy between magnetic and dielectric fields, at such frequencies of the impressed voltage as to make the total impedance or admittance a minimum. To supply the resonating circuit with the oscillatory energy necessitates a transient starting period during which the amplitude of each oscillation is greater than the one preceding. For systems having constant finite circuit constants in which the resonance phenomena reach permanent values, the growth of the transient follows the exponential law. This increase in the magnitude of the oscillations during the starting period is illustrated by the oscillograms in Figs. 139 and 140. In these oscillograms the power supply was cut off when the resonance had reached the permanent stage. The decay parts of the oscillograms in Figs. 139 and

FIG. 139.—Cumulative resonance. Initial and decay transients. R = 16.4 ohms; L = 89 millihenrys; C = 0.25 microfarads; timing wave 100 cycles; f = 1070 cycles; decrement = 0.084.

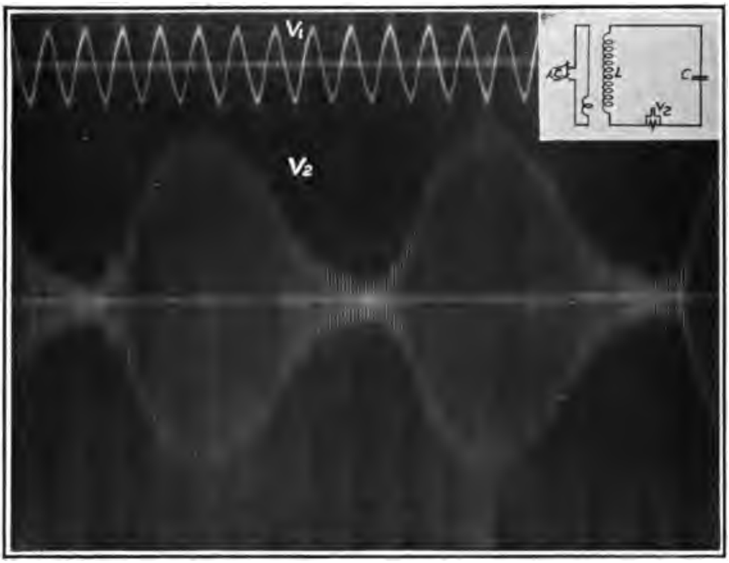

FIG. 140.—Resonance in high speed signaling.
R = 10 ohms; L = 89 millihenrys; C = 0.25 microfarads; timing wave 100 cycles; frequency = 1070 cycles; decrement = 0.052.

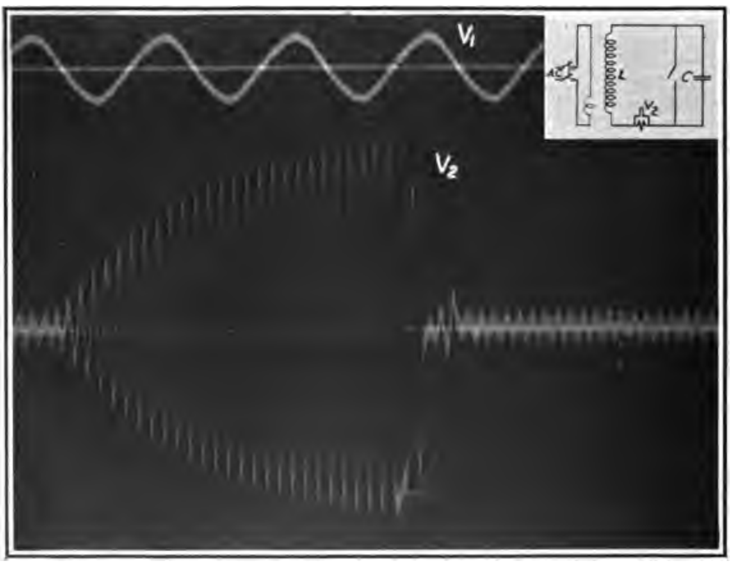

FIG. 141.—Resonance limited by spark gap discharge.
R = 15 ohms; L = 89 millihenrys; C = 0.25 microfarads; timing wave 100 cycles; frequency = 1070 cycles; decrement = 0.079.

140, represent, therefore, free oscillations with a decrease in amplitude as the electric energy is dissipated into heat.

In Fig. 141 the starting period is of the same form as in Fig. 139 or 140, but not the decay stage. It is evident from the circuit connections that the decay of the resonating currents or voltages will differ in shape depending at what instant in the cycle the short circuit occurs. The oscillogram in Fig. 141, for which the short circuit was produced by spark-over, occurred near the maximum point of the voltage wave with practically all of the oscillating energy initially stored in the dielectric field of the condenser.

Problems and Experiments

1. Take oscillograms showing the transients accompanying the growth and decay of cumulative resonance in circuits similar to Figs. 139, 140 and 141.

2. Take oscillograms of the transient oscillations of two inductively coupled circuits similar to Figs. 133, 134 and 135.

3. Take oscillograms of the transient oscillations in two dielectrically coupled circuits similar to Figs. 136, 137 and 138.

CHAPTER IX

OSCILLOGRAMS

In the preceding chapters the fundamental principles of electric transient phenomena are illustrated by a number of oscillograms, many of which the student should reproduce in order to gain the necessary appreciation of the quantitative value of the factors involved. However, the laboratory work in the course should not be restricted to the reproduction of oscillograms appearing in the text for which quantitative data are provided, or to the taking of other oscillograms that merely illustrate the fundamental principles. For while the gaining of clear concepts of the basic laws of transient electric phenomena is of primary importance, training in applying the principles to practical engineering problems is likewise an essential part of the work. Ample material for this purpose is available in all electrical engineering laboratories. The oscillograms in this chapter, Figs. 142 to 161, which were selected from the laboratory reports of students in the introductory course in electric transients, may be taken as typical examples. The students were required to outline the problem, to select the necessary apparatus and instruments, to make preliminary calculations and to predict the form and shape of the transients to be recorded. They made all the adjustments on the oscillograph, obtained experimentally the recorded quantitative data, took the oscillograms, developed the films and prepared a report on the transients photographically recorded by the oscillograph. Each oscillogram represents a separate problem to be analyzed on the basis of the principles discussed in the preceding chapters.

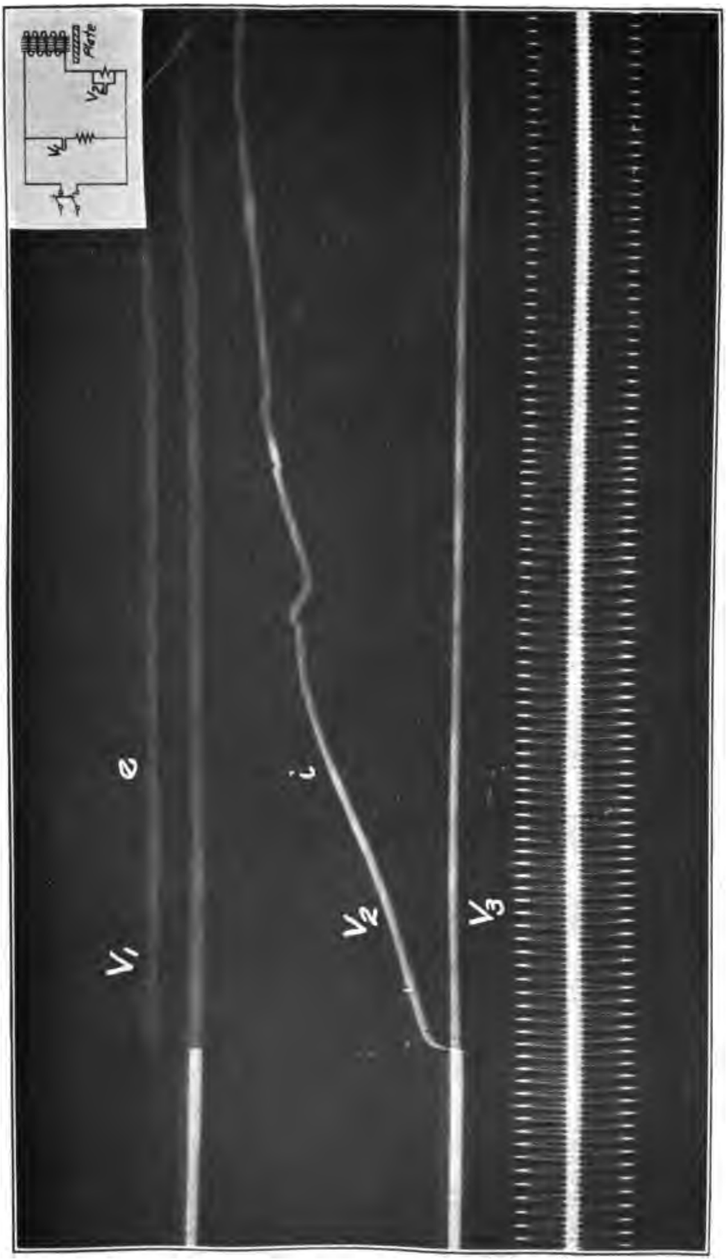

FIG. 142.—Starting transients of a D. C. lifting magnet. E = 105 volts d.c.; I = 1.56 amps.; V_1 = voltage; V_2 = current; V_3 = 100 cycle timing wave.

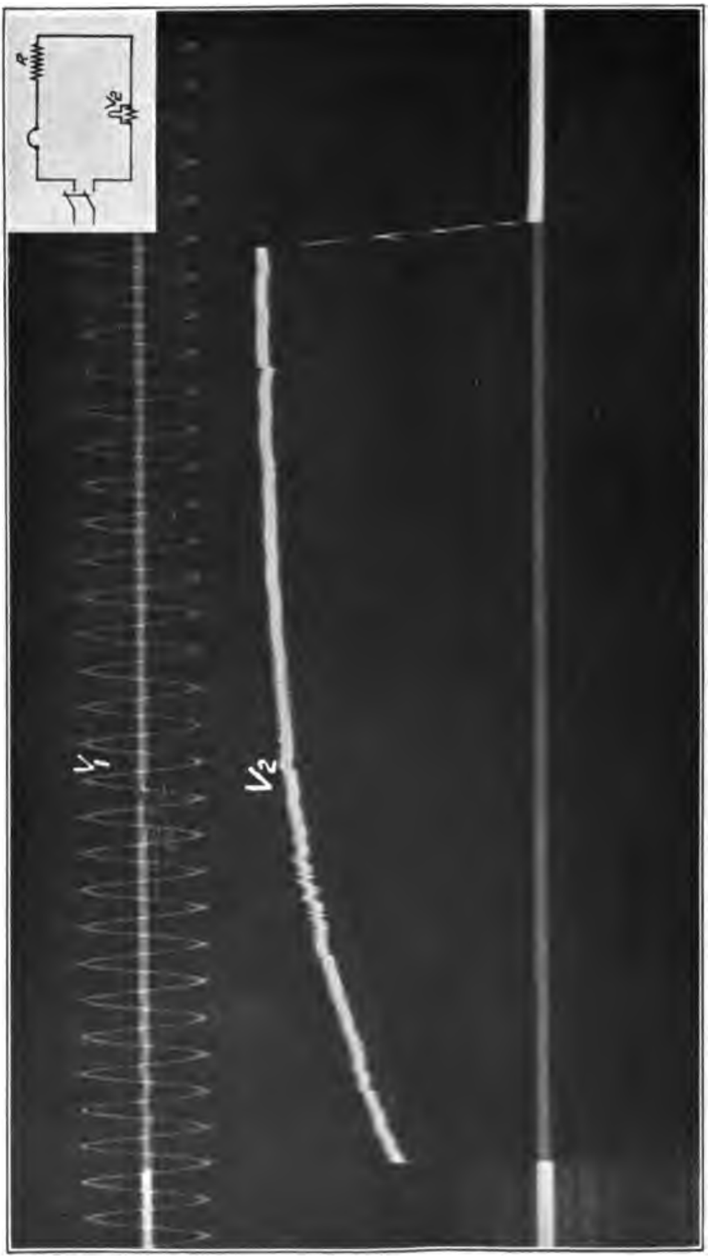

FIG. 143.—Opening of a D.C. circuit breaker due to overload. E = 125 volts; I = 10 amps.; V_1 = 100 cycle timing wave; V_2 = current.

FIG. 144.—Opening of an A.C. circuit breaker due to overload. E = 120 volts; I = 15 amps.; V_1 = 100 cycle timing wave; V_2 = current.

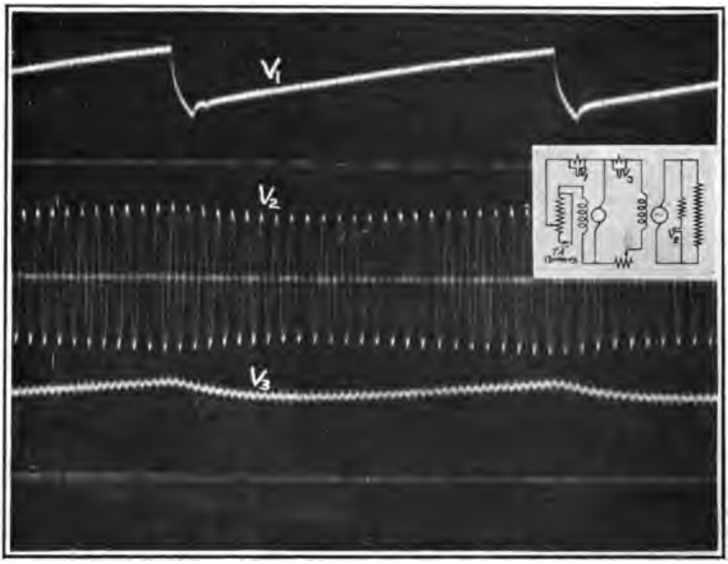

FIG. 145.—T. A. regulator operating transients.
V_1 = exciter field current; V_2 = alternator field current; V_3 = alternator terminals.

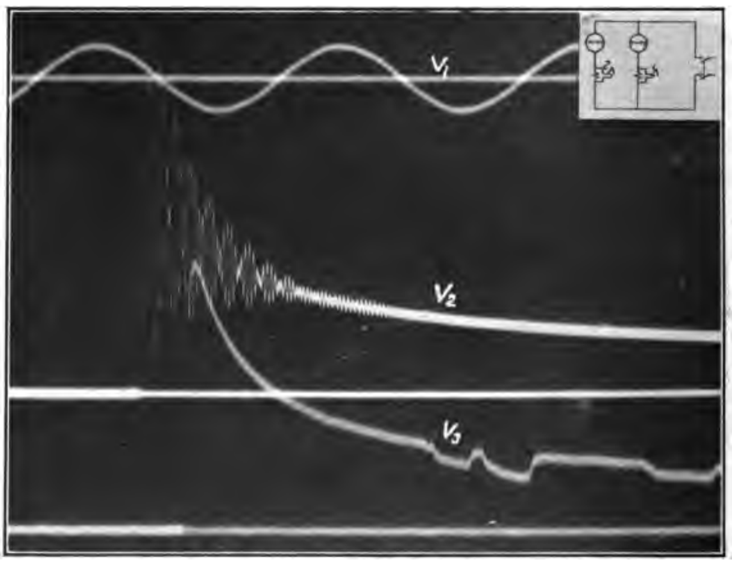

FIG. 146.—Undamped oscillograph vibrator oscillations.
V_1 = timing wave, 100 cycles; V_2 = Oscillations of undamped oscillograph vibrator superimposed on tungsten lamp starting transient. V_3 = starting transient (vibrator damped) of tungsten lamp, imperfect contact.

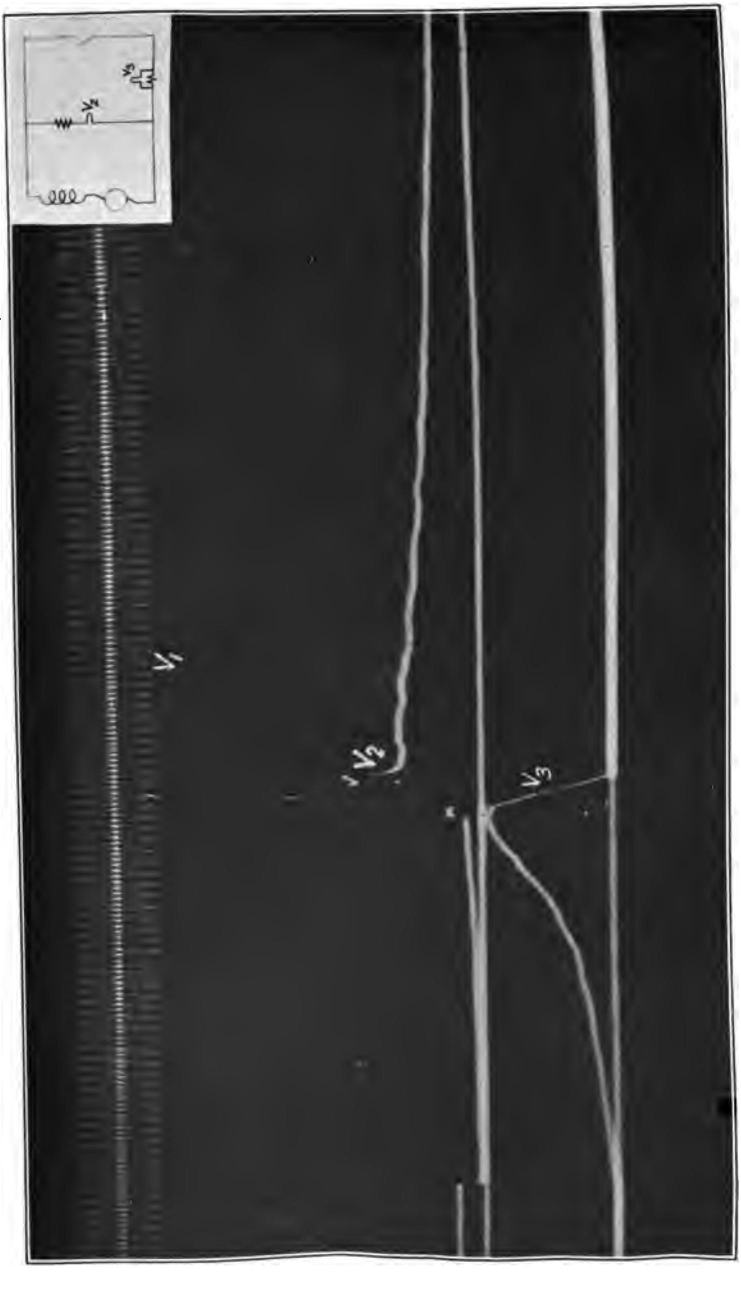

FIG. 147.—Short circuit on a series generator. V_1 = 100 cycle timing wave; V_2 = total voltage; V_3 = current.

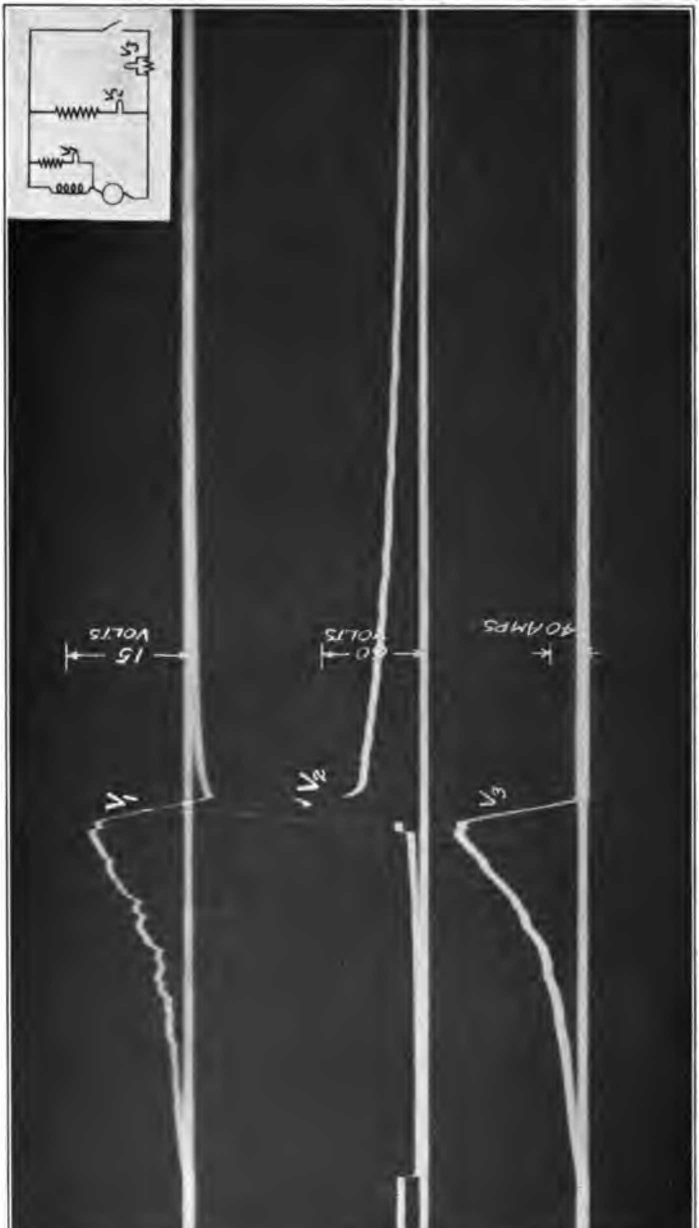

FIG. 148.—Short circuit on a series generator. V_1 = series field voltage; V_2 = total voltage; V_3 = current.

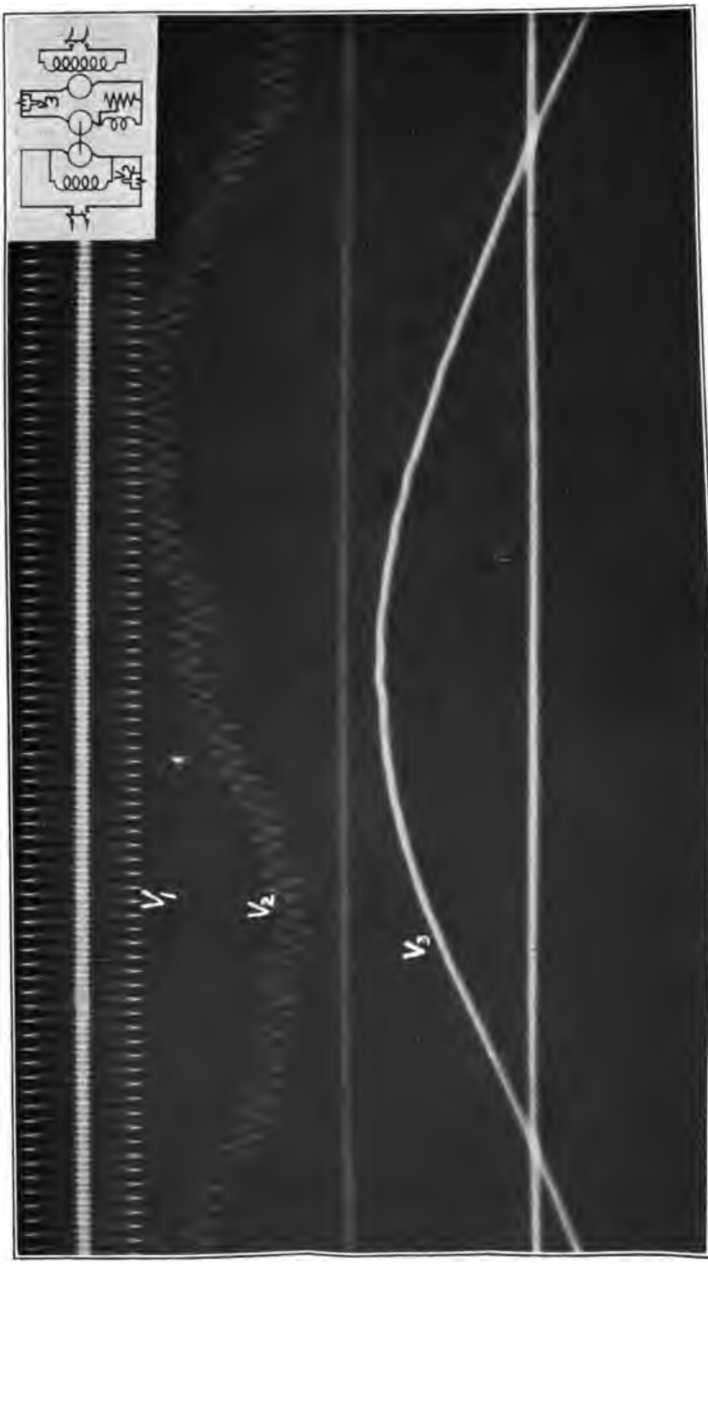

FIG. 149.—"Bucking broncho." V_1 = 60 cycle timing wave; V_2 = motor armature current; V_3 = series generator current; maximum motor current = 8 amps.; maximum generator current = 45 amps.; frequency = 60 cycles.

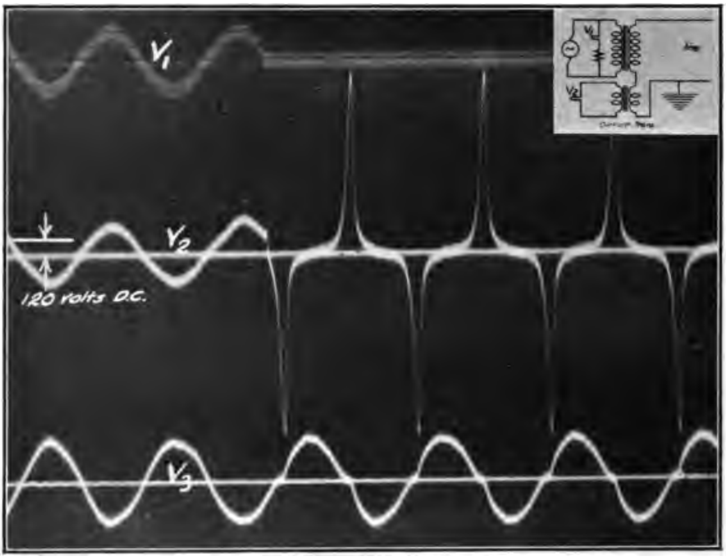

FIG. 150.—Current transformer transients.
V_1 = secondary current; V_2 = secondary voltages; V_3 = primary current; primary I = 60 amps.; secondary I = 3.5 amps.; core undersaturated before transient.

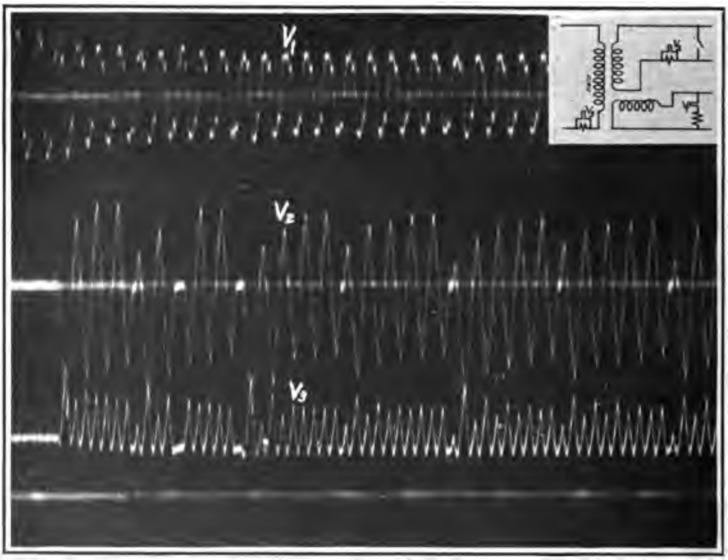

FIG. 151.—Single phase short circuit on a two-phase alternator.
Open phase voltage = 605 volts; short circuit current = 23 amps.; E, field = 500 volts; I, field = 3.25 amps.; frequency = 60 cycles; V_1 = open phase voltage; V_2 = short circuit current; V_3 = field current; brushes sparking.

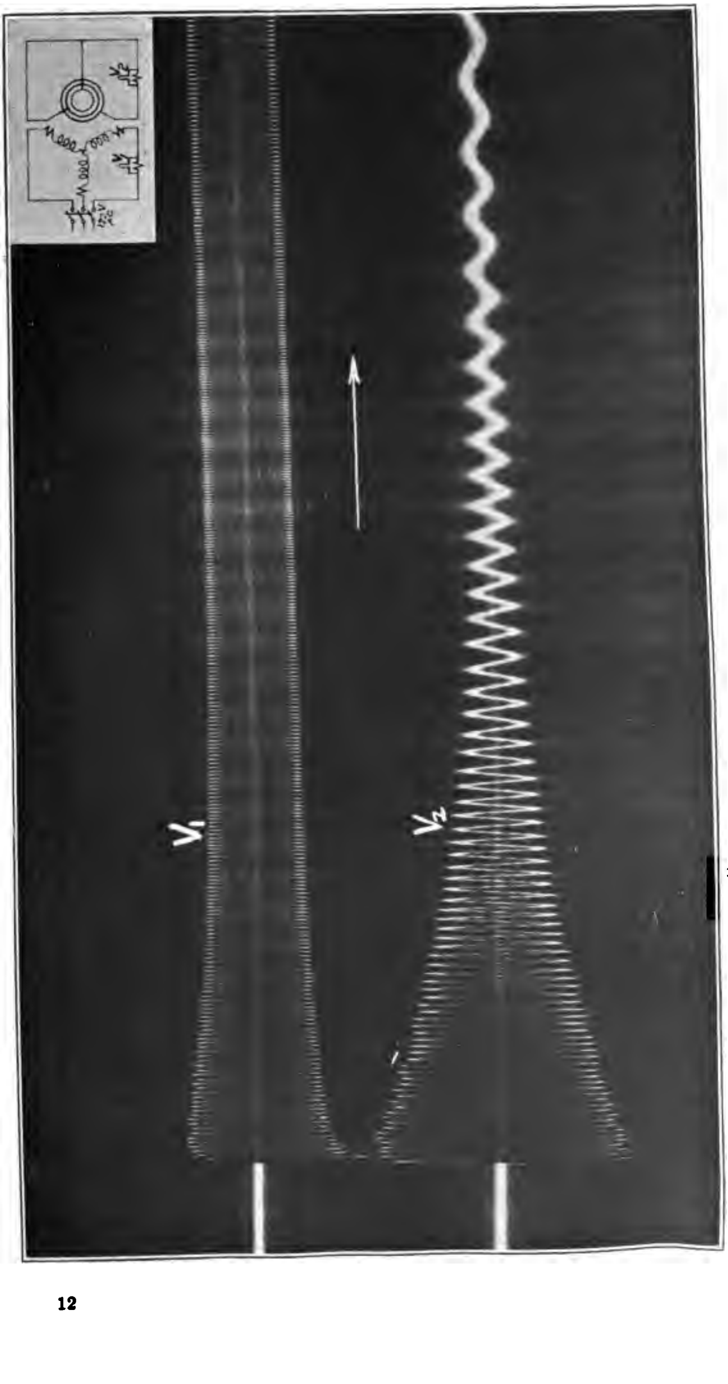

Fig. 152.—Starting transient on a three phase induction motor without external secondary resistance. E = 120 volts; I, primary = 22.2 amps.; I, secondary = 2.1 amps.; V_1 = stator current; V_2 = rotor current.

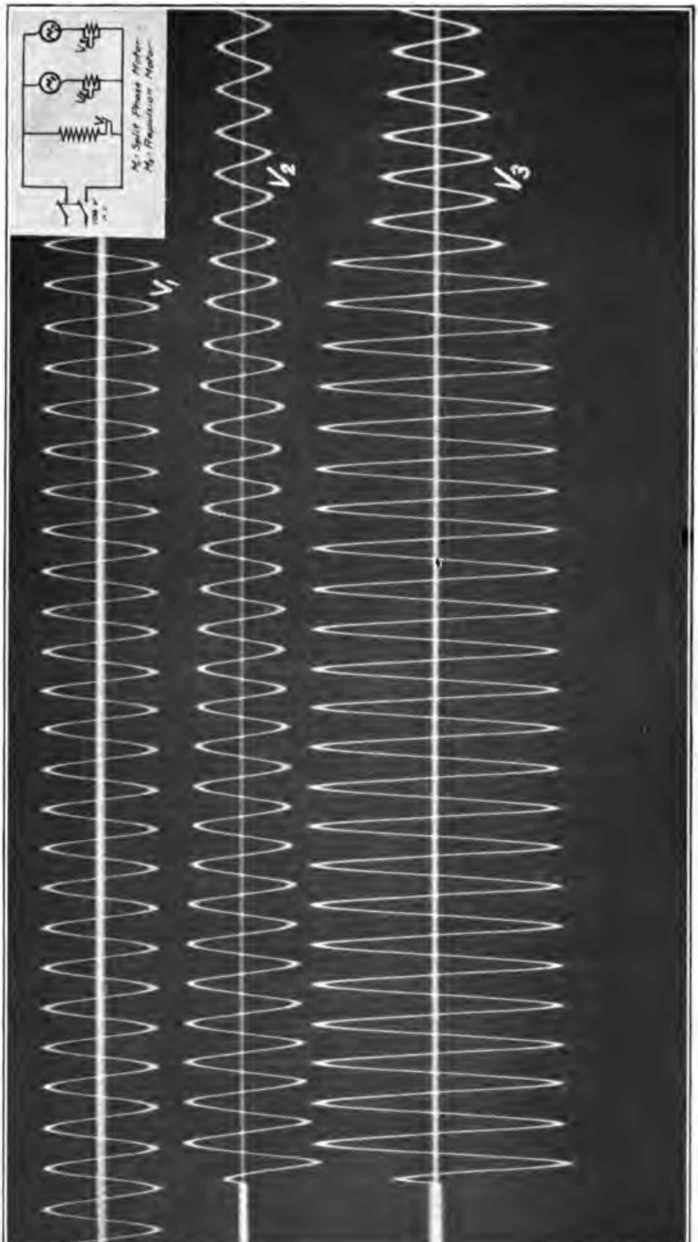

FIG. 153.—Starting transients of a repulsion-induction motor and a split phase motor. E = 106 volts; V_2 = 2 amps. sustained; V_3 = 2.8 amps. sustained; M_1 = split phase motor; M_2 = Repulsion motor.

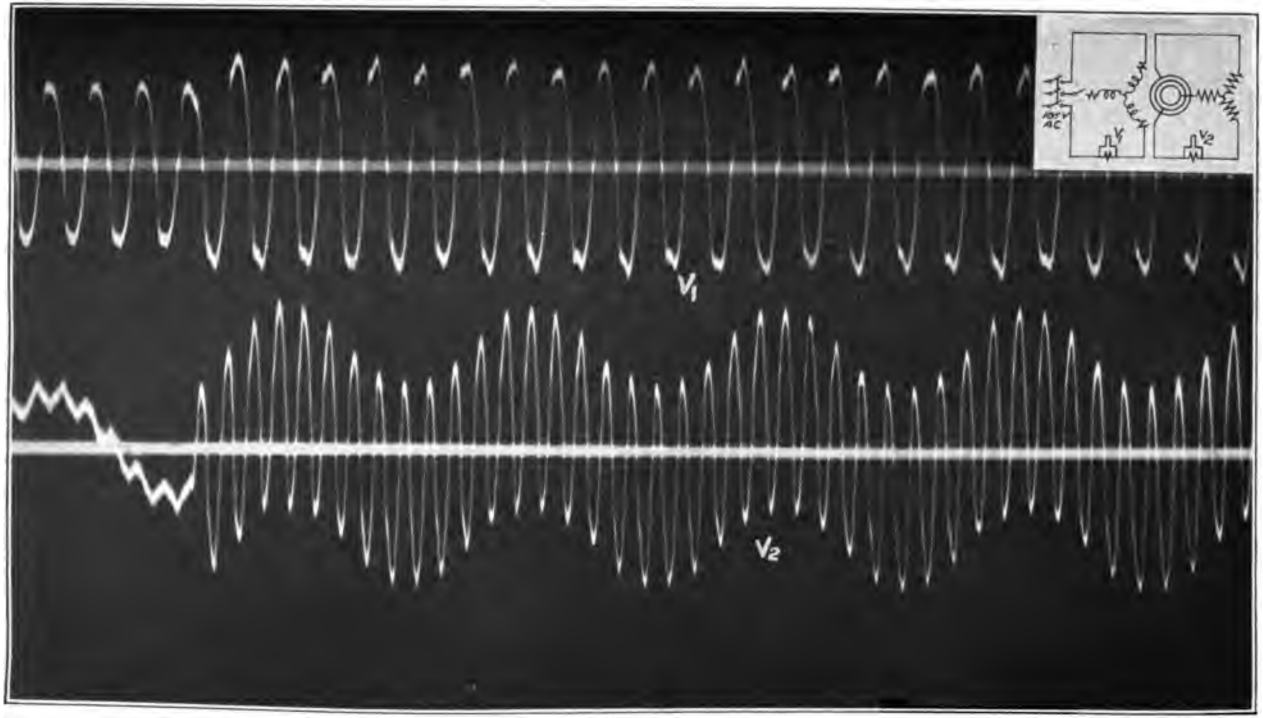

FIG. 154.—Single-phase operation of a three-phase induction motor. E = 105 volts; I, primary = 15.8 amps. with switch closed; I, secondary = 4.2 amps. with switch closed; I, secondary = 12.0 amps.; with switch open; V_1 = primary current; V_2 = secondary current.

FIG. 156.—Short circuit on a rotary converter. D.C. breaker opens first; $E_{D.C.} = 123$ volts; $I_{A.C.} = 16$ amps. $I_{D.C.} = 15$ amps.; V_1 = D.C. voltage; V_2 = A.C. current; V_3 = D.C. current.

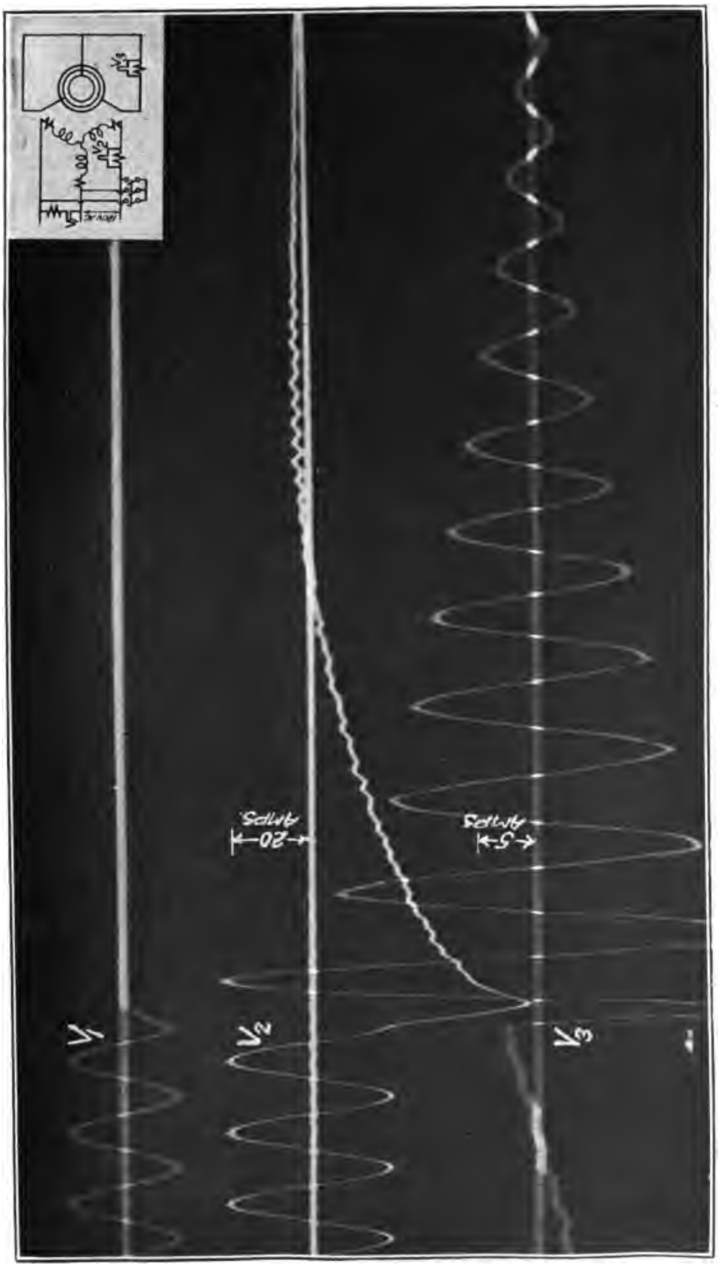

FIG. 155.—Transients in a three-phase induction motor due to short circuit in stator. E = 120 volts; V_2 = primary current; V_3 = secondary current; frequency = 57 cycles.

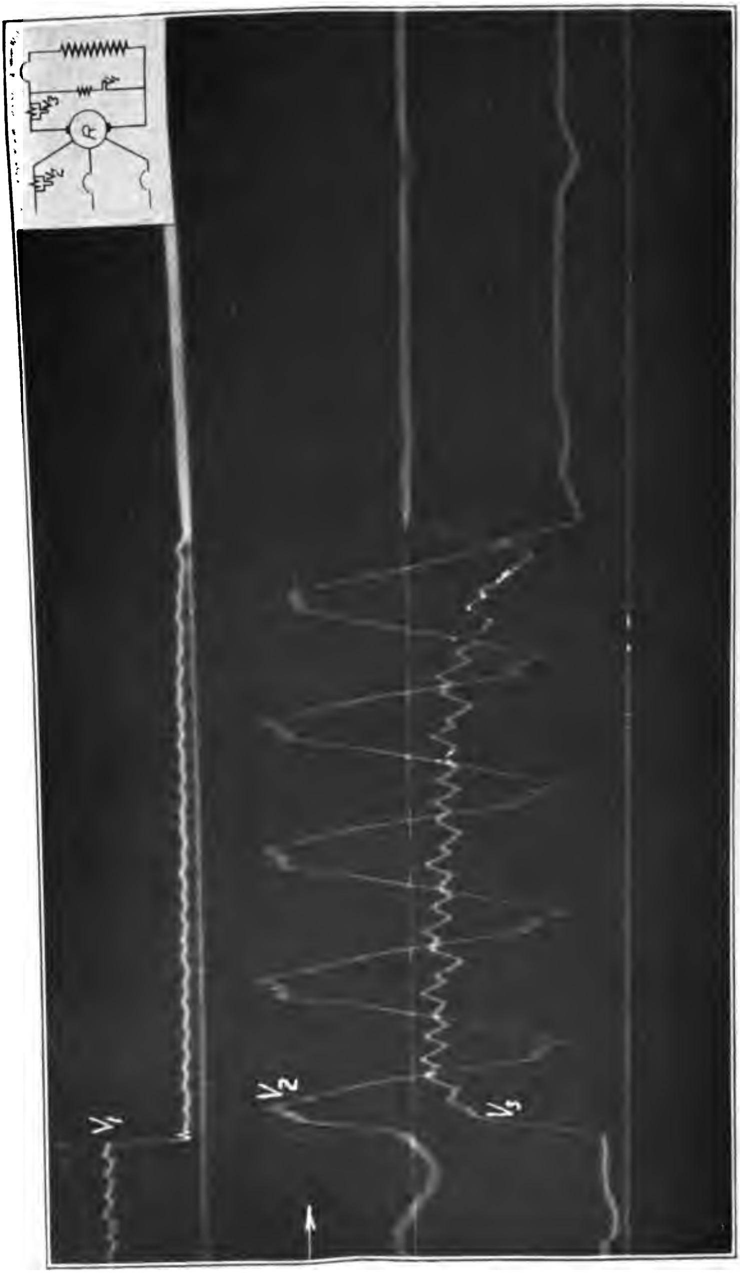

FIG. 157.—Short circuit on a rotary converter. D.C. breaker does not open. $E_{D.C.} = 123$ volts; $I_{A.C.} = 16$ amps.; $I_{D.C.} = 15$ amps.; V_1 = D.C. voltage; V_2 = A.C. current; V_3 = D.C. current.

FIG. 158.—Short circuit on a rotary converter. A.C. breaker does not open; $E_{D.C.}$ = 123 volts; $I_{A.C.}$ = 16 amps.; $I_{D.C.}$ = 15 amps.; V_1 = D.C. voltage; V_2 = A.C. current; V_3 = D.C. current.

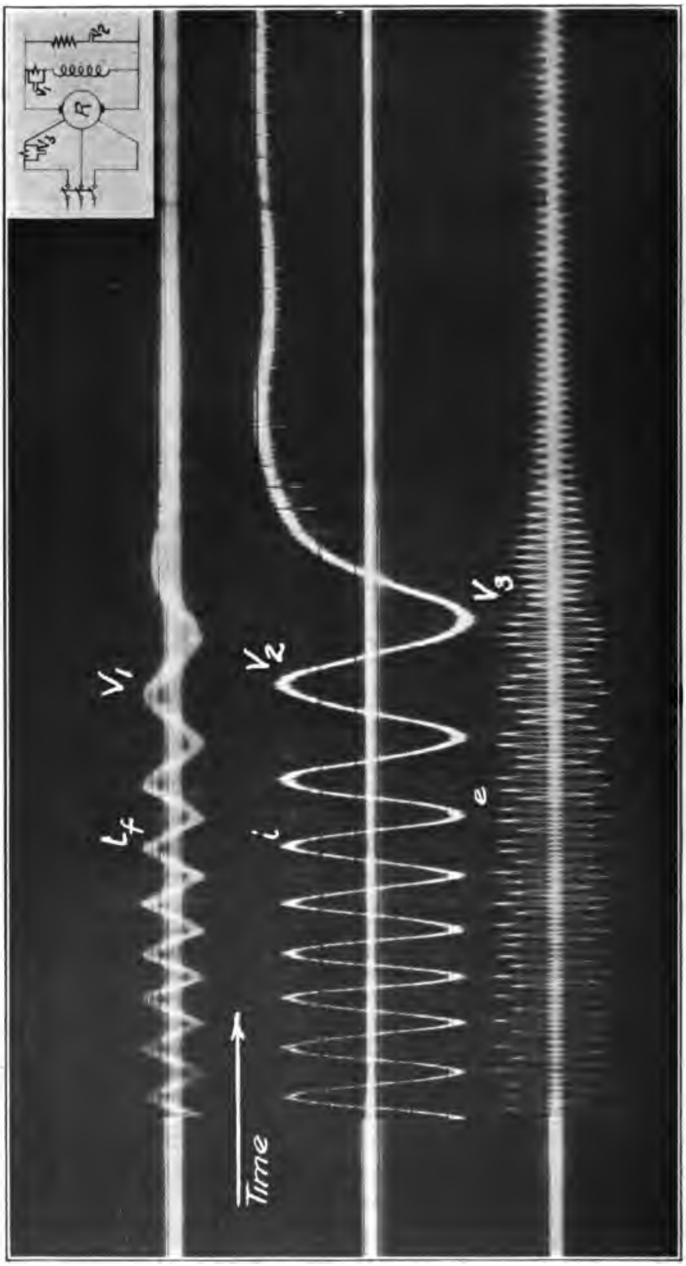

FIG. 159.—Synchronizing a rotary converter from 85 per cent synchronous speed. E = 250 volts D.C.; V_1 = field current; V_2 = D.C. voltage; V_3 = synchronizing current.

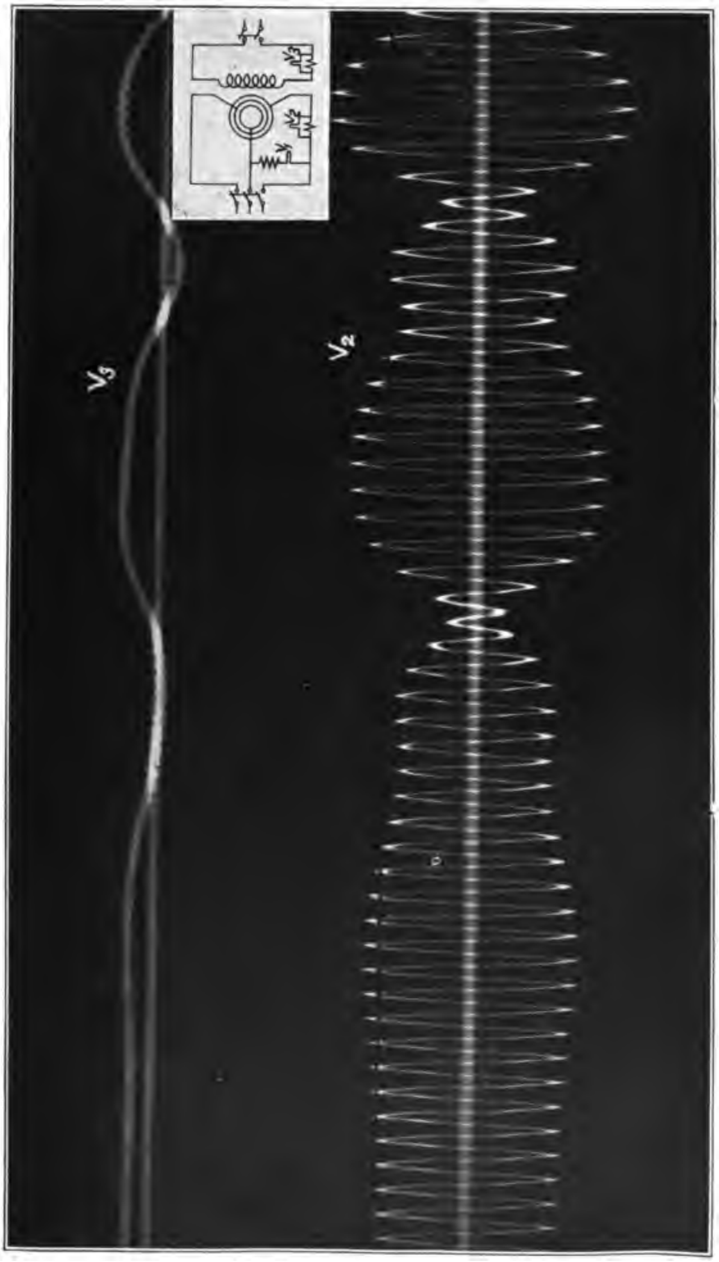

FIG. 160.—Synchronous motor falling out of step due to overload. E = 90 volts; I, arm., = 28 amps.; I, field = 1.2 amps.; V_1 = voltage; V_2 = armature current; V_3 = field current.

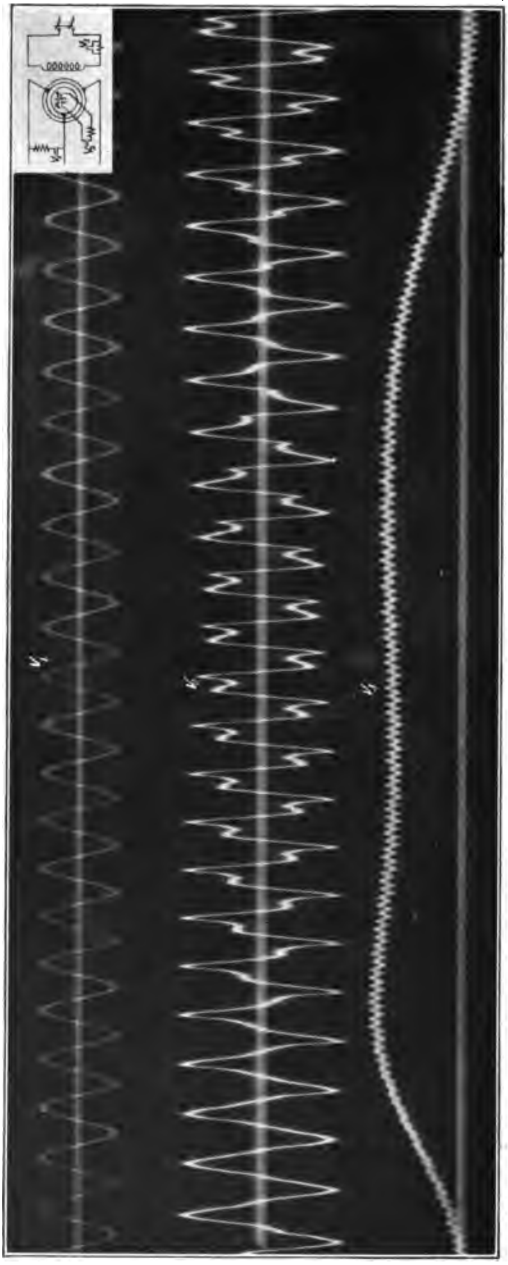

FIG. 161.—Flux distribution of a synchronous motor when slipping a pole. Terminal voltage 100 volts; pilot coil voltage = 0.4 volts; I = 5 amps.; V_1 = 60 cycle wave; V_2 = pilot coil voltage; V_3 = field current.

Problems and Experiments

1. Take oscillograms of a number of transients in circuits of the types shown in this chapter. In each case obtain quantitative data and prepare a report giving an explanation of the transients appearing in the oscillogram based on the fundamental principles of transient electric phenomena.

2. Find several electric transients in the laboratory under different circuit conditions from those described in the book. For each case draw diagrams of the proposed circuit connections showing the location of the vibrators; make preliminary calculations as to the amount of resistance required in each vibrator circuit; the most desirable speed of the film drum, etc., to give a well proportioned oscillogram; take the oscillogram; record the quantitative data; develop the film and make prints. Compare the predicted forms of the curves with the photographic record and check the preliminary calculations with the final circuit data. Prepare a report on the transients recorded on the oscillogram.

APPENDIX

Developing and Printing Oscillograms.—The finished oscillogram, even if perfect electrically, is often disappointing photographically. Care and cleanliness in the manipulation of the photographic film and printing paper will reduce these failures to a negligible quantity.

Starting with the unexposed film, the photographic process will be traced to the completed print, ready for the files. Cleanliness is essential. During no part of the process should the hands come in contact with the sensitized side of the negative. In order to accomplish this, the film and its black protecting paper should be placed on the drum as a unit, with the black paper on the outside. After the film and paper have been adjusted to the proper position, the paper may be removed from the drum. In this way the hands have not touched the surface of the film.

Unlike most photographic work, the permissible time of exposure for oscillograms is limited, especially in high speed work. Stray light of any nature is injurious. For this reason it is highly desirable to load the film-holders in complete darkness and to develop for the first two or three minutes without even the ruby light. After a little practice the student will have no trouble in working without the darkroom light.

Any metol-hydrochinon film developer may be used with varying degrees of success. Where only a few negatives are made at odd times, Eastman's "Special" developer is satisfactory. This developer will give better results if some of the used developer be added to the fresh solution. In our laboratories the following stock solution is used: water 64 oz., metol one drachm, hydrochinon one-half oz., sodium sulphite 2 oz., sodium carbonate 3 oz., potassium bromide 30 grains. This stock solution is diluted in the proportion of two parts stock solution to one part water.

It is very important that the developer be used at a temperature of 65 deg. F. The hydrochinon is inactive at lower temperatures, resulting in slow development and a flat negative which lacks density and contrast. If used at a higher temperature, the negative will gain density rapidly but will be lacking in contrast and show a decided tendency to fog in the unexposed portions.

The exposed negative should be given maximum development possible without fogging the unexposed portions. The image should be allowed to develop until it appears quite definite on the reverse side of the negative. A good rule to follow is to develop until by comparison with the back of the negative, the sensitized side appears quite gray. The gray tone will disappear in the fixing bath and further development is detrimental.

Care should be taken to fix and wash the negatives properly. The film should be left in the standard fixing bath at least five minutes longer than is necessary to dissolve the last visible trace of un-reduced silver salts. After careful fixing, the film should be washed for at least twenty minutes in running water. It is desirable to rinse off the surface with a tuft of cotton before hanging up to dry. The hurry which often comes in the completion of the day's work in the laboratory, results in haste in the darkroom. If the fixing and washing processes are slighted, the film, though apparently good at the time, becomes worthless in a few months on account of staining.

The same developer may be used for the printing paper, except that it should be always mixed fresh just before using. The best results are obtained by following the printed instructions accompanying the photographic paper.

In order to get the maximum contrast in the finished print, it is necessary to use the most contrasting photographic paper. The paper which has proven the best is the Eastman "Azo," grade No. 4, glossy, although others may satisfy the individual user. If this is purchased in ten yard rolls, twenty inches wide and cut on a circular saw or

band-saw to four and one-half inch widths, four small rolls result with a two inch roll left over for use in testing exposure.

Prints should be given normal exposure so that, with normal or full development the background reaches good density without appreciable reduction of the silver in the highlights. As usual, prints should be fixed fifteen minutes in a standard hypo bath and washed for at least twenty-five minutes in running water. The best finish is obtained by drying the prints on ferro-type plates, which imparts high gloss to the surface.

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